[80]

Seat No._______ No. of Printed Pages : 2

SARDAR PATEL UNIVERSITY

B.Sc.SEM-V EXAMINATION

18^{th} November 2019 , Monday

10.00 a.m. to 01.00 p.m.

US05CMTH04 (Abstract Algebra-I) Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) A nonempty subset H of finite group G is a subgroup of G iff ....
(a) $a-b \in H$ (b) $a+b \in H$ (c) $(ab)^{-1} \in H$ (d) $ab \in G$

(2) In group G , $(aba^{-1})^{-1} =$ ....
(a) aba^{-1} (b) $a^{-1}b^{-1}a$ (c) $ab^{-1}a^{-1}$ (d) $a^{-1}ba$

(3) Every infinite cyclic group has exactly .... generators .
(a) 3 (b) 1 (c) 2 (d) 4

(4) In Klein 4-group $G' = \{e, a, b, c\}$, $b^2 = \ldots$
(a) e (b) b (c) c (d) a

(5) .... is generator of group $\{\pm 1 , \pm i\}$.
(a) 2 (b) -1 (c) 1 (d) -i

(6) If H is any normal subgroup of G then ....
(a) Hx=Hy (b) Hx = xH (c) Hx = H (d) xH = yH

(7) Every cyclic group of order 4 is isomorphic to ......
(a) Klein 4-group (b) Z (c) N (d) Z_4

(8) Order of S_4 is ....
(a) 3 (b) 12 (c) 24 (d) 4

(9) Every group of order ....is abelian group .
(a) 2 (b) 5 (c) 4 (d) 6

(10) A permutation σ is said to be odd permutation if signature of σ is ....
(a) 2 (b) -1 (c) 1 (d) -2

Q.2 Attempt the short questions. **[any ten]** [20]

(1) Prove that intersection of two subgroups of a group G is also a subgroup of G.

(2) Find inverse of each element of $(Z_7^*, \cdot)$

(3) Prove that identity of group is unique.

(4) Give an example of finite cyclic group.Verify it .

(5) State Euler's theorem.

(6) Find all right cosets of $-3\mathbb{Z}$ in $\mathbb{Z}$.

(7) Define order of element in group G. Find $O(\bar{3})$ in Z_6.

(8) Let $\theta : G \to G'$ be a homomorphism.Then prove that $Ker\theta$ is a subgroup of G.

(9) Prove that $\theta : Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z.

(10) Let$G = \{e, a, b, c\}$ be the Klein 4-group,$H = \{e, a\}$, $K = \{e, b\}$.Show that $G = H \times K$.

(PTO)

(11) Prove that product of two permutation need not be commutative.

(12) Express the following permutation as a product of disjoint cycles

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 7 & 6 & 8 & 1 \end{pmatrix}$$

Q.3(a) Check whether the sets $(Q-\{1\},*)$, where $*$ is defined as $a*b=a+b-ab\ ,\forall\, a,b\in Q-\{1\}$ forms a group or not. Verify it. [5]

(b) Prove that a non-empty subset H of a group G is subgroup iff $ab^{-1}\in H\ \forall\ a,b\in H.$ [5]

OR

Q.3(c) Let G be a semigroup. Assume that, for all a,b$\in G$, the equations $ax=b$ and $ya=b$ have unique solutions in G. Then prove that G is a group. [5]

(d) Let H and K be subgroups of group G. Then prove that HK is subgroup of G iff $HK=KH$. [5]

Q.4(a) Prove that every subgroup of an infinite cyclic group is also an infinite cyclic group. [5]

(b) Let G be a group and $a,b\in G$ such that $ab=ba$. If $O(a)=n, O(b)=m$ with m,n relatively prime, then prove that $O(ab)=mn$. [5]

OR

Q.4(c) Let G be a finite cyclic group of order n, then prove that G has $\phi(n)$ generators [5]

(d) Let H be any subgroup of group G. Then prove that [5]

(i) $aH=H\ \Leftrightarrow a\in H$

(ii) $aH=bH\ \Leftrightarrow b^{-1}a\in H$

Q.5(a) Let $G'=\{1,\rho,\rho^2,...,\rho^{n-1}\}$ be the multiplicative group of n^{th} root of unity, where $\rho=e^{2\pi i/n}$. Then prove that $Z_n\simeq G'$. [5]

(b) Let $\theta : G\to G'$ be a homomorphism. Then prove that [5]

(i) $Ker\theta$ is a subgroup of G (ii) $Ker\theta$ is a normal subgroup of G.

OR

Q.5(c) State and Prove second isomorphism theorem. [5]

(d) Prove that a subgroup H is normal in group G iff $xH=Hx\ \forall\, x\in G$. [5]

Q.6(a) Let $G=H\times K$ be external direct product of H and K, then prove that $G/K'\simeq H$, where $K'=\{(e_H,k)/k\in K\}$. [5]

(b) Prove that the set S_n of all permutation on n symbols forms a non-commutative group. [5]

OR

Q.6(c) State and prove Cayley's theorem. [7]

(d) Show that $G=Z_2\times Z_2$ is the Klein 4-group. [3]

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