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SARDAR PATEL UNIVERSITY

B.Sc. SEM- V EXAMINATION

13^{th} November 2019 , Wednesday

10.00 a.m. to 01.00 p.m.

US05CMTH02

(Real Analysis-II)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) A function is continuous if.....
(a) $\lim_{(x,y)\to(a,b)} f(x,y) \neq f(a,b)$ (b) $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$
(c) $\lim_{(x,y)\to(a,b)} f(x,y) = f(0,0)$ (d) none of these

(2) The function $f(x,y)$ has an extreme value at point (a,b) if...
(a) $f(x,y) = f(a,b)$ (b) $f_x(a,b) = 0 = f_y(a,b)$ (c) $f_x(a,b) \neq 0$
(d) $f_x(a,b) > 0$

(3) A number ξ is said to be a limit point of a sequence $\{S_n\}$ if every neighbourhood of ξ contains .... of members of the sequence.
(a) finite number (b) infinite number (c) one (d) at least one

(4) A series $\sum u_n$ is convergent then ...
(a) $\lim_{n\to\infty} u_n = 1$ (b) $\lim_{n\to\infty} u_n \neq 0$ (c) $\lim_{n\to\infty} u_n = 0$
(d) $\lim_{n\to\infty} u_n$ does not exists

(5) The limit points of sequence $\{(-1)^n(2+\frac{1}{n}) : n \in N\}$ are....
(a) 2 and -2 (b) 0 and 2 (c) 2 only (d) not exists

(6) If $f(x,y) = 2x^3 + 3y^2$ then $f_{yyy} = ...$
(a) $6y$ (b) $2x^3$ (c) 6 (d) 0

(7) A positive term series $\sum \frac{1}{n^p}$ is convergent iff:
(a) $p = 1$ (b) $p < 1$ (c) $p > 1$ (d) $p = 0$

(8) $\lim_{x\to1}\lim_{y\to-1} \dfrac{4x^3y^2}{x^2+y^2} =$
(a) -2 (b) 3 (c) 2 (d) 1

(9) A sequence $\{1,0,2,0,3,0,4,0,\dots\}$ has ...... as a limit point.
(a) 1 (b) 2 (c) 0 (d) not exists

(10) A stationary point is called saddle point of function f if it is...
(a) extreme point (b) not an extreme point (c) $f_x(a,b) = 0$
(d) none of these

Q.2 Attempt **any ten** in short: [20]

(1) Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $lim u_n = 0$.

(PTO)

(2) State necessary condition for $f(x,y)$ to have an extreme value at (a,b).

(3) State D'Alembert's Ratio test.

(4) Define: Neighbourhood of a point of two variable.

(5) Define first order partial derivatives. If $f(x,y) = 2x^2 - xy + 2y^2$, then find f_x and f_y at point $(1,2)$.

(6) By using definition of limit, show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

(7) Prove that every convergent sequence is bounded.

(8) Prove that $(x,y) \overset{lim}{\to} (0,0) \frac{2xy^2}{x^2+y^4}$ does not exists.

(9) Prove that $(x,y) \overset{lim}{\to} (4,\pi)\ x^2 \sin\frac{y}{x} = 8\sqrt{2}$.

(10) Test for convergence the series $\sum \frac{1}{n^{1+\frac{1}{n}}}$.

(11) If $\sum u_n = u$ and $\sum v_n = v$, then prove that $\sum(u_n + v_n) = u + v$.

(12) Define: Repeated limits and continuity of a function of two variables.

Q.3(a) State and prove Bolzano-Weierstrass theorem for sequences. [6]

(b) Show that every convergent sequence is bounded and has a unique limit. [4]

OR

Q.3(c) Show that the necessary and sufficient condition for the convergence of a sequence $\{S_n\}$ is for each $\epsilon > 0$, there exist a positive integer m such that $|s_{n+p} - s_n| < \epsilon$, $\forall n \geq m \wedge p \geq 1$. [6]

(d) If $n \overset{lim}{\to} \infty a_n = a$ and $a_n \geq 0$, then prove that $a \geq 0$. [4]

Q.4(a) If $\sum u_n$ and $\sum v_n$ are two positive term series such that $n \overset{lim}{\to} \infty \frac{u_n}{v_n} = l$, where l is anon-zero finite number, then show that the two series converges or diverges together. [6]

(b) If $\sum_{n=1}^{\infty} u_n = u$, then prove that $\sum_{n=0}^{\infty} u_n = u + u_0$ and $\sum_{n=2}^{\infty} u_n = u - u_1$. [4]

OR

Q.4(c) State and prove Cauchy's root test. [6]

(d) Show that the series $\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \cdots + \frac{1}{(\log n)^p} + \ldots$, diverges for $p > 0$.[4]

Q.5(a) Show that for the functions $f(x,y) = \begin{cases} x\sin(\frac{1}{y}) + y\sin(\frac{1}{x}), & xy \neq 0 \\ 0, & xy = 0 \end{cases}$ [6]

limit exist at the origin but the repeated limits do not.

(b) Show that $f(xy, z-2x) = 0$ satisfies, under suitable conditions, the equation $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2x$. What are these conditions? [4]

OR

Q.5(c) Prove that, by the transformation $u = x - ct$, $v = x + ct$, the partial differential equation $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ reduces to $\frac{\partial^2 z}{\partial u \partial y} = 0$. [6]

(d) Show that the function

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous at the origin. [4]

Q.6(a) State and Prove Taylor's theorem. [6]

(b) Prove that the first four terms of the Maclaurin expansion of $e^{ax} \cos by$ are [4]

$$1 + ax + \frac{a^2x^2 - b^2y^2}{2!} + \frac{a^3x^3 - 3ab^2xy^2}{3!}.$$

OR

Q.6(c) Investigate the maxima and minima of the function [6]

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy.$$

(d) Expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$. [4]

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