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SARDAR PATEL UNIVERSITY

B.Sc. SEM- V EXAMINATION

11^{th} November 2019 , Monday

10.00 a.m to 1.00 p.m

Sub.: Mathematics (US05CMTH01: Real Analysis-I)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answer book. [10]

(1) The continuous function on closed interval is.....
(a) not bounded (b) open set (c) bounded (d) none

(2) The limit point of $\{\frac{(-1)^n}{n+1} : n \in N\}$ is
(a) 0 (b) 1 (c) 2 (d) not exists

(3) The set $\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, ...\}$ is...
(a) open and closed set (b) open set (c) closed set (d) neither open nor closed

(4) The field which does not have the least upper bound property is ...
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ (d) $\mathbb{R}$

(5) Every uniformly continuous function is.....
(a) continuous (b) not continuous (c) unbounded (d) none

(6) A function f is said to have a removable dicontinuity at $x = c$ if...
(a) $\emptyset$ (b) $lim f(x) \neq f(c)$ (c) $lim f(x) = f(c)$ (d) None.

(7) The Set $\cup(\frac{1}{n}, 1 - \frac{1}{n})$ is
(a) $(0, 1)$ (b) $\{0, 1\}$ (c) $[0, 1]$ (d) $[0, 1)$

(8) The derived set of $S = \{1, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}, \frac{1}{3^4}, ...\}$ is....
(a) S (b) Q (c) ϕ (d) R

(9) If $f'(c) < 0$, then function f is....... at c.
(a) not derivable (b) increasing (c) descreasing (d) none of these

(10) The infinite intersection of open set is......
(a) open and closed set (b) not open set (c) closed set (d)all of these.

Q.2 Attempt **any ten** in short: [20]

(1) Prove that limit of a function is unique, if exists.

(2) Prove that $a^n = a.a.a......a$ (n times).

(3) Let $m = inf(S)$, where S is a bounded set. Show that $m \in$ closure of S .

(4) Find the supremum and the infimum of a set $\{1 + \frac{(-1)^n}{n} : n \in N\}$.

(5) Prove that $|x| = max(x, -x)$.

(6) Prove that the superset of a neighbourhood(nbd) of a point x is also a nbd of x.

(7) If f is derivable at point c then, show that function $1/f$ derivable at point c, where $f(c) \neq 0$.

(8) Determine whether closure of the set $[2, 50] \cup (0, 1) \cup Q$ is closed or not.[PTO]

(P.T.O.)

(9) Show that the function $f(x) = \frac{1}{x}$ is uniformly continuous on $(0, 1]$.
(10) In usual notations, prove that $\lim_{x \to a}(f+g)(x) = l + m$.
(11) Is $(S \cap T)' = S' \cap T'$? Justify your answer.
(12) Prove that supremum of a set S of numbers, if it exists, is unique.

Q.3(a) Prove that the set of rational numbers is not order complete. [5]
(b) Show that there is no rational number whose square is 11. [5]

OR

Q.3(c) In usual notations, prove that $E(x) = e^x$ for all $x \in R$. [5]
(d) If a be a positive real number and b any real number then there exists a positive integer n such that $na > b$. [5]

Q.4(a) Prove that there exists a positive number π such that $C(\pi/2) = 0$ and $C(x) > 0$ for $0 \leq x < \pi/2$. [5]
(b) Show that every bounded infinite set has the smallest and the greatest limit point. [5]

OR

Q.4(c) State and Prove Bolzano-Weierstrass theorem for a set. [5]
(d) Show that the derived set of a set is closed. [5]

Q.5(a) Show that a function $f : [a, b] \to \mathbf{R}$ is continuous at point c of $[a, b]$ iff $\lim_{n \to \infty} c_n = c \Rightarrow \lim_{n \to \infty} f(c_n) = f(c)$. [5]
(b) Examine the function for continuity at $x = 0, 1$ and 2. Also discuss the kind of discontinuity where $f(x)$ defined on $\mathbf{R}$ by [5]

$$f(x) = \begin{cases} -x^2, & \text{if } x \leq 0 \\ 5x - 4, & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$$

OR

Q.5(c) In usual notations, prove that $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{l}{m}$, provided $m \neq 0$. [5]
(d) If f is continuous on $[a, b]$ then prove that it attains its bounds at least once in $[a, b]$. [5]

Q.6(a) Show that $log(1 + x)$ lies between $\frac{x}{1+x}$ and x for all $x > 0$. [5]
(b) Show that a function which is continuous on a closed interval is uniformly continuous on that interval. [5]

OR

Q.6(c) State and Prove Darboux's theorem for derivable function. [5]
(d) Define uniform continuity. Prove that the function x^2 is uniformly continuous on $[-1, 1]$. [5]

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