[21] SEAT No.________ No. of Printed Pages : 03

Sardar Patel University

B.Sc.- Sem-V Mathematics

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US05CMTH05

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Time: 3 hours **Total: 70 marks**

Q-1 MCQs 10.00 am to 01.00 pm [10]

1. LCM of the two numbers 25 & 30 is______
   - A. 5
   - B. 25
   - C. 30
   - D. 150

2. Which of the following number is composite number?
   - A. 11111
   - B. 111111
   - C. 111
   - D. 11

3. If sum of all positive divisors of a is denoted by $S(a)$ then, $S(20) =$ ___
   - A. 6
   - B. 20
   - C. 42
   - D. 26

4. If μ denotes the Mobious function, then $\mu(12) =$ _____
   - A. 0
   - B. 1
   - C. -1
   - D. None of the above

5. $(a, b, c) =$ _____
   - A. (a, b)
   - B. (b, c)
   - C. $((a, b), c)$
   - D. $(a + b, c)$

6. If $(a, b) = 1$ then there exist $x, y \in Z$ such that $ax + by =$ _____
   - A. 1
   - B. 0
   - C. a
   - D. b

7. $3^{80} \equiv$ _____$(mod\ 5)$
   - A. 0
   - B. 1
   - C. 3
   - D. 5

8. What is 10^{th} Fibonacci number?
   - A. 34
   - B. 55
   - C. 89
   - D. 21

(P.T.O)

9. Which of the following number is divisible by 3?

A. 1234
B. 8451
C. 35488
D. 8454

10. If $2x \equiv 5(mod\ 7)$ then $x =$______

A. 5
B. 3
C. 2
D. 55

Q-2 Attempt any Ten short questions [20]

1. Find, gcd(525,231).
2. Write the formula for $P(a)$, the product of all positive divisors of a.
3. Define: Fermat's number.
4. Define: Fibonacci number.
5. Define: Reduced Residue System modulo m.
6. State Euclidean algorithm.
7. If $a|bc$ & $(a, b) = 1$ then prove that $a|c$.
8. Find number of multipliers of 7 among integers from 200 to 500.
9. Find $2(50!)$.
10. Define equivalent relation.
11. What is $\phi(625)$?
12. State Chinese theorem.

Q-3 (a) State and prove fundamental theorem of divisibility. 05

(b) If $(a, b) = d$ then prove that there exists $x, y \in Z$ such that $ax + by = d$. 05

OR

(a) Prove that $[a, b](a, b) = a \cdot b$ if $a \cdot b > 0$ 05

(b) If p is prime number then prove that there exist no positive integer a, b such that $a^2 = pb^2$. 05

Q-4 (a) Prove that every prime factor of Fermat's number $F_n (n > 2)$ is of the form $2^{n+2}t + 1$ for some $t \in Z$. 05

(b) In usual notation prove that $(a) < a\sqrt{a}, \forall a > 2$. 05

OR

(a) If $a > 1$ then prove that $\sum_{d/a} \mu(d) = 0 = \sum_{d/a} \mu\left(\frac{a}{d}\right)$ 05

(b) In usual notation prove that, 05

$$u_{m+n} = u_{m-1}u_n + u_m u_{n+1} \forall m, n \in N$$

Q-5 (a) State and prove the necessary and sufficient condition for a positive integer n can be divided by 3. 05

(b) Prove that positive integer solution of $x^{-1} + y^{-1} = z^{-1}, (x, y, z) = 1$ is of the form $x = a(a + b), y = b(a + b), z = ab$ where $a, b > 0, (a, b) = 1$. 05

OR

(a) Prove that the integer solution of $x^2 + 2y^2 = z^2, (x, y) = 1$ can be expressed as, $x = \pm(a^2 - 2b^2), y = 2ab, z = a^2 + 2b^2$. 05

(b) Find positive integer solution of following equation: 05
$7x + 9y = 213$

Q-6 (a) Find all positive integers m & n such that $\phi(mn) = \phi(m) + \phi(n)$. 05

(b) Let $m \equiv 0(mod\ 2)$. If $a_1, a_2, \dots, a_n$ & $b_1, b_2, \dots, b_n$ are CRS modulo m then prove that $a_1 + b_1, a_2 + b_2, \dots, a_m + b_m$ is not CRS modulo m. 05

OR

(a) If $(a, m) = d\ (d > 1)$ then prove that $ax + b \equiv 0(mod\ m)$ has solution if and only if d/b. Also prove that it has $'d'$ solutions $x_i = a + i\frac{m}{d}(mod\ m)$ where $i = 0,1,2, \dots, d - 1$. In which $x \equiv a\left(mod \frac{m}{d}\right)$ is unique solution of $\frac{a}{d}x + \frac{b}{d} \equiv 0\left(mod \frac{m}{d}\right)$ 05

(b) Solve the following system of congruent equations 05
$x \equiv 2(mod\ 3), x \equiv 3(mod\ 5), x \equiv 2(mod\ 7)$

—X—