[24] SEAT No.________ No. of Printed Pages : 03

SARDAR PATEL UNIVERSITY

BSc Sem V Examination

Mathematics

US05CMTH04-Abstract Algebra-I

10|04|2019

Date : 10-04-2019 Wednesday Time : 10-00 TO 1-00 PM

Q.1 Answer the following by selecting correct choice from the options. (10)

1. The order of -1 in the multiplicative group of non zero rational numbers is________

A. Infinite B. 1
C. 2 D. 3

2. The order of the group S_3 is ______

A. 1 B. 4
C. 3 D. 6

3. Every infinite cyclic group has exactly ________ generators.

A. 0 B. 2
C. 3 D. None

4. The multiplicative inverse of 6 in Z_7^* is _________

A. 6 B. 2
C. 3 D. None

5. The Generators of cyclic grou $G = \{All\ 4^{th}\ roots\ of\ unity\}$ under multiplication is ______.

A. ± 1 B. $\pm i$
C. 1 D. None

6. The order of an even permutation group A_4 is _______.

A. 24 B. 6
C. 12 D. 1

7. If $H = 3\mathbb{Z}$ is a subgroup of additive group $G = \mathbb{Z}$ then the index $(G : H) =$ _____

A. 1 B. 4
C. 3 D. 6

(P.T.O.)

8. The inverse of an element a in the group $(\mathbb{Q}-\{-1\},\ *)$ where $*$ is defined by $a*b=a+b-ab, \forall a,b \in \mathbb{Q}-\{-1\}$ is _______

A. $-a$ B. a

C. $\frac{a}{a-1}$ D. $\frac{a}{a+1}$

9. If $\emptyset$ is Euler's function then $\emptyset(6)=$ ______.

A. 4 B. 3

C. 2 D. 1

10. If G is a group of order 10 then possible order of its subgroups is ______.

A. 5 B. 3

C. 4 D. 7

Q.2 Answer any TEN. **(20)**

1) In group G, prove that every element of group Ghas unique inverse.
2) Is union of two subgroups of a group again a subgroup? Justify your answer.
3) Prove that G is commutative $iff\ (ab)^2=a^2b^2 \quad , \ \forall\ a,b \in G$.
4) Is every cyclic group an abelian? Justify.
5) Let H be a subgroup of group G then prove that $Ha=Hb \Leftrightarrow ab^{-1} \in H$.
6) Let $f: G \to G'$ be any homomorphism then prove that $f(a^{-1})=(f(a))^{-1}$, G and G' are groups and $a \in G$.
7) Define : Quotient group.
8) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 6 & 5 & 1 \end{pmatrix}$ as a product of disjoint cycles.
9) Define Transposition. Express the cycle $(1\quad 2\quad 4\quad 5\quad 3)$.
10) Prove that the Kernal of Homomorphism $f, Ker\ f$ is a subgroup of group G.
11) State second isomorphism theorem.
12) Prove that the product of two permutations need not be commutative.

Q. 3 (a) State and prove the necessary and sufficient condition for a non empty subset H Of a group G to be a subgroup of G. (5)

(b) Check whether the set $(\mathbb{Z}_6, +)$ forms a group or not. Is it commutative? (5)

OR

Q. 3 (a) State and prove cancellation laws for group. (5)

(b) Prove that the centre $\mathbb{Z}(G)$ of group G is a subgroup of group G. (5)

Q. 4 (a) State and prove Lagrange's theorem. (5)

(b) Let G be a group and $a, b \in G$ such that $ab = ba$. If $O(a) = n, O(b) = m$ with m, n are relatively prime, then prove that $O(ab) = mn$. (5)

OR

Q. 4 (a) Prove that any two right cosets of H in G have the same number of elements. (5)

(b) Let G be a finite cyclic group of order n, then prove that G has $\emptyset(n)$ generators.(5)

Q. 5 (a) State and prove Third isomorphism theorem. (6)

(b) Prove that the kernel of homomorphism$Kerf$ is normal subgroup of group G. (4)

OR

Q. 5 (a) Define Normal subgroup. Prove that a subgroup H is Normal in group G iff (6)

$xH = Hx, \ \forall\, x \in G$.

(b) Prove that Homomorphic image of cyclic group is also cyclic. (4)

Q. 6 (a) State and prove Cayley's theorem. (6)

(b) (i) $O(A_n) = \frac{n!}{2}$ (4)

(ii) Express the inverse of cycle $(1 \quad 2 \quad 4 \quad 5 \quad 3)$ as a product of transpositions.

OR

Q. 6 (a) Prove that G is direct product of subgroups H and K iff (7)

(i) every $x \in G$ can be uniquely expressed as $x = hk, h \in H, k \in K$.

(ii) $hk = kh, h \in H, k \in K$.

(b) Let $G = \{e, a, b, c\}$ be the Klein 4-group, $H = \{e, a\}, K = \{e, b\}$. Show that (3)

$G = H \times K$.

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