[17]

SEAT No.________ No. of Printed Pages : 03

**Sardar Patel University**

**Sem-V - Mathematics**

09|04|2019, Tuesday

**US05CMTH03**

**Time: 3 hours** - 10.00 a.m to 1.00 P.M **Total: 70 marks**

**Q-1** MCQs [10]

1. If $d$ is discrete metric then what is $d(20,25)$?
   A. 5 B. 0
   C. 1 D. $-5$

2. Which of the following $d: R \times R \to R$ is not metric on $R$?
   A. $d(x,y) = |x-y|$ B. $d(x,y) = |x^2 - y^2|$
   C. $d^*(x,y) = \min\{1, d(x,y)\}$ D. None

3. In $M = [0,1]$ with usual metric, $B\left[\frac{1}{4}, \frac{1}{2}\right] =$______
   A. $\left[0, \frac{3}{4}\right]$ B. $\left(0, \frac{3}{4}\right)$
   C. $\left(\frac{3}{4}, 0\right)$ D. $\left[\frac{3}{4}, 0\right]$

4. Let $A = [0,1]$ then which of the following subset of $A$ is not open in $A$
   A. $\left(\frac{1}{2}, 1\right]$ B. $\left[\frac{1}{2}, 1\right)$
   C. $\left(\frac{1}{2}, \frac{2}{3}\right)$ D. None of the above

5. The set of all cluster points of $(0,1)$ is _____
   A. $(0,1]$ B. 0
   C. 1 D. $[0,1]$

6. Which of the following set is not closed?
   A. $\{0\}$ B. $\phi$
   C. $\left\{\frac{1}{n} : n \in N\right\}$ D. $R$

7. In metric space, arbitrary union of closed set is_____
   A. closed B. open
   C. both A & B D. None

8. Which of the following subset of $R$ with discrete metric is dense?
   A. $N$ B. $Z$
   C. $Q$ D. None

(P.T.O.)

9. Every finite subset of metric space is______
   - A. open
   - B. totally bounded
   - C. dense
   - D. connected

10. Which of the following function from $R$ to $R$ with usual metric is not uniformly continuous?
    - A. $\sin x$
    - B. $\cos x$
    - C. $\tan x$
    - D. $x+1$

**Q-2** Attempt any ten short questions **[20]**

1. Define: Metric Space.
2. Define: convergent sequence in metric space.
3. Define: Cluster point in metric space.
4. Is the set $A=\{10,20,30,40,50\}$ closed in $R$ with usual metric? Justify.
5. Define: Connected set.
6. Which are the bounded sets in $R$ with discrete metric?
7. Define: Totally bounded set.
8. Give an example of a set which is bounded but not totally bounded.
9. Define: $\epsilon-$ dense set.
10. Define: Bounded function.
11. Define: Uniformly continuous function.
12. Give an example of function which is not uniformly continuous.

**Q-3 (a)** Let $\rho: R^2 \times R^2 \rightarrow R$ defined by $\rho(P,Q)=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. Show that $\rho$ is metric on $R^2$. **05**

**(b)** Let $f$ and $g$ be real valued functions defined on metric space $(M,\rho)$. If $f$ and $g$ are continuous at $a \in M$ then prove that $f \cdot g$ is also continuous at $a$. **05**

**OR**

**(a)** Let $(M_1,\rho_1),(M_2,\rho_2)$ and $(M_3,\rho_3)$ be metric spaces. Let $f: M_1 \rightarrow M_2$, $g: M_2 \rightarrow M_3$. If $f$ is continuous at $a \in M_1$ and $g$ is continuous at $f(a) \in M_2$ then prove that $gof$ is continuous at $a$. **05**

**(b)** Prove that every function from $R$ to $R$ with discrete metric is continuous. **05**

**Q-4 (a)** Let $(M,\rho)$ be a metric space. Then prove that any open sphere **05**

in $M$ is an open set.

**(b)** Let $(M,\rho)$ be a metric space and let $A$ be a proper subset of $M$. Then the subset $G_A$ of $A$ is an open subset of $(A,\rho)$ is and only if there exist an open subset $G_M$ of $(M,\rho)$ such that $G_A = A \cap G_M$. 05

**OR**

**(a)** If $E$ is any subset of the metric space $M$, then show that $\bar{E}$ is closed. 05

**(b)** Is the intersection of an infinite number of open sets is open? Justify. 05

**Q-5 (a)** A subset $A$ of $(R,d)$ is totally bounded if and only if $A$ contains only a finite number of points. Where $d$ is discrete metric. 05

**(b)** State and prove generalised nested interval theorem. 05

**OR**

**(a)** Let $M$ be a metric space. Then prove that $M$ is connected if and only of every continuous characteristic function on $M$ is constant. 05

**(b)** Prove that every contraction mapping is continuous. 05

**Q-6 (a)** Let $f$ be a continuous function from the compact metric space $M_1$ into the metric space $M_2$. Then the range $f(M_1)$ of $f$ is also compact. 05

**(b)** Show that $f: R \to R$ defined by $f(x) = x$ is uniformly continuous. 05

**OR**

**(a)** Let $f$ be a one-one continuous function on a compact metric space $(M_1,\rho_1)$ onto $(M_2,\rho_2)$. Then prove that $f^{-1}$ is continuous and hence $f$ is homeomorphism of $M_1$ onto $M_2$. 05

**(b)** Give an example of a function which is one-one, onto, continuous but its inverse is not continuous. 05

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