[20] SEAT No.______ No. of Printed Pages : 03 SC

Sardar Patel University , Vallabh Vidyanagar

B.Sc. Examinations :2018-19

Subject : Mathematics US05CMTHO2 [Real Analysis-II] Max.Marks: 70

Date : 08/04/19, Monday Time:10:00A.M. TO 01:00P.M.

1. Answer the following by selecting correct choice from the options : **[10]**

(1) If the sequence $\{s_n\}$ is bounded then it ____.

(a) Oscillates infinitely (b) Oscillates finitely
(c) Diverges to $+\infty$ (d) Diverges to $-\infty$

(2) A sequence $\{s_n\}$ is strictly increasing if for all n ,_____.

(a) $s_{n+1} \geq s_n$ (b) $s_{n+1} \leq s_n$
(c) $s_{n+1} < s_n$ (d) $s_{n+1} > s_n$

(3) The Range of sequence is always ______ set.

(a) empty set (b) infinite set

(c) non- empty set (d) none of these

(4) If $\{S_n\}$ is a sequence of partial sums of the series,then S_4 = ______.

(a) $u_1 + u_2 + u_3 + u_4$ (b) $u_1 + u_2 - u_3 - u_4$

(c) $u_1 - u_2 + u_3 - u_4$ (d) $u_1 \times u_2 \times u_3 \times u_4$

(5) A positive term series $\sum \frac{1}{n^p}$ is convergent iff _____.

(a) p = 1 (b) $p < 1$

(c) $p > 1$ (d) $p < 0$

(6) For infinite series $\sum u_n$ if $\lim_{n \to \infty} u_n = 0$,then series______.

(a) Converges always (b) Converges some times

(c) Does not converge (d) None of these

(7) $\lim_{x \to 1} \lim_{y \to -1} \frac{4x^3 y^2}{x^2 + y^2}$ =______.

(a) 1 (b) 3

(c) -2 (d) 2

(P.T.O.)

(8) A function is said to be continuous in a region if it is continuous at ______ of the given region.

(a) Only one point (b) Every point

(c) Some point (d) Nowhere

(9) The extreme value of f(a,b) is called minimum if sign of f(x,y) - f(a,b) is _____.

(a) Positive (b) Negative

(c) Alternate +ve & -ve (d) None of these

(10) A Stationary point is called point of function f it is ______.

(a) extreme point (b) not extreme point

(c) f_x (a,b) = 0 (d) non of these

2. Answer any TEN of the following. [20]

1) Define: Convergence of sequence.
2) If $\{a_n\}$ and $\{b_n\}$ are two sequences then prove that $\lim_{n \to \infty}(a_n + b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$
3) Prove that $\lim_{n \to \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.
4) Define : Infinite series.
5) Prove that a positive term series converges iff the sequence of its partial sum is bounded above.
6) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.
7) Prove that $\lim_{(x,y) \to (0,0)} \frac{x\sin(x^2+y^2)}{x^2+y^2} = 0$.
8) Define : Repeated limits.
9) If f(x ,y) = 3 $x^2 - 2xy + y^2$ then find f_x and f_y at the point (-1,2).
10) State Maclaurin's theorem .
11) Define :Extreme Value.
12) Prove that f(x ,y) = $x^2y + x^4 + y^2$ has a minimum at (0,0).

3. (a) State and Prove Bozano-Weierstrass theorem for sequence. [5]

(b) Prove that $\lim_{n \to \infty} \sqrt[n]{n} = 1$. [5]

OR

3. (a) State and Prove Cauchy's first theorem on limits. [5]

(b) Prove that every convergent sequence is bounded and has a unique limit. [5]

4. (a) State and Prove comparision test of first type. [5]

(b) Prove that the series $\frac{1}{(log2)^p} + \frac{1}{(log3)^p} + \dots\dots\dots \frac{1}{(logn)^p} + \dots$ diverges , p> 0. [5]

OR

4. (a) State and Prove D'Alembert's Ratio test. [5]

(b) If $\lim_{n \to \infty} a_n$ = a and $a_n \geq 0$ then prove that a ≥ 0. [5]

5. (a) Define Limit of a function and by using the definition of limit prove that [5]

$$\lim_{(x,y) \to (1,2)} 3xy = 6.$$

(b) Prove that the function , $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous at [5]

the origin.

OR

5. (a) If V is a function of two variables x and y and x =r cosθ ,y =r sinθthen prove that [5]

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}.$$

(b) Show that for given function limit exist at the origin but the repeated limit [5]

does not , $f(x,y) = \begin{cases} \frac{x^2+y^2}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$

6. (a) State and Prove Taylor's theorem. [5]

(b) Find maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$. [5]

OR

6. (a) Prove that the first four terms of the Maclaurin's expansion of

$e^{ax} cosby$ are $1 + ax + \frac{a^2x^2 - b^2y^2}{2!} + \frac{a^3x^3 - 3ab^2xy^2}{3!}$. [5]

(b) Show that $2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0,0). [5]

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