[10-D] SEAT No.________ No. of Printed Pages : 04

**Sardar Patel University , Vallabh Vidyanagar**

**B.Sc. Examinations :2018-19**

**Subject : Mathematics** **US05CMTHO1 [Real Analysis]** **Max.Marks: 70**

**Date :** 5/4/2019 Friday **Time:10:00A.M. TO 01:00P.M.**

Q.1 Choose the correct option for each of the following. [10]

(1)Which of the following is not an ordered field ?

(a) Q (b) R (c) C (d) none of these

(2) The greatest number of a set ,if exists is ....

(a) the supremum of the set (b) infimum of the set

(c) not unique (d) none

(3) The supremum of $\left\{\frac{1}{n} / n \in N\right\}$ is ....

(a) 0 (b) 1 (c) 2 (d) none

(4) In $\left(0,\frac{\pi}{2}\right)$ function S(x) is ....

(a) strictly increasing (b) strictly decreasing

(c) stationary (d) none

(5) The derived set of (0,1) is ....

(a) [0,1] (b) (0,1] (c) [0,1) (d) (0,1)

(6) The interior of Z is

(a) N (b) Z (c) Ø (d) R

(7) If $\lim_{x \to a} f(x)$ exists but f(a) is not defined then f is said to have a discontinuity of ....

(a) first type (b) second type

(c) removable type (d) first type from left

(P.T.O)

(8) The continuous function on closed interval is ....

(a) not bounded (b) open set

(c) bounded (d) none

(9) The condition that f is strictly decreasing at c is .....

(a) $f'(c) < 0$ (b) $f'(c) > 0$

(c) $f'(c) = 0$ (d) none

(10) Which of the following functions is not derivable at 0?

(a) $|x|$ (b) sinx (c) cosx (d) tanx

Q.2 Attempt any Ten. [20]

(1) Define : A Complete Ordered Field.

(2) Prove that supremum of a set S of numbers,if it exists ,is unique.

(3) Find the g.l.b and l.u.b of $\left\{1 + \frac{1}{n^3} / n \in N\right\}$ if they exist.

(4) Prove that every open interval is an open set.

(5) Define: An interior point of a set.

(6) Porve that C(-x) = C(x) ,∀ x ϵ R.

(7) Find $\lim_{x \to 0} \frac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$ if it exists.

(8) Define : Limit of a function .

(9) Give an example of a function having discontinuity of first kind at some point.

(10) Examine whether the function $f(x) = \begin{cases} x, if\ 0 \le x < 1 \\ 1, if \quad x \ge 1 \end{cases}$ is differentiable at x = 1 or not.

(11) Prove that a function which is uniformly continuous on

an interval is continuous on that interval.

(12) Define derivable function at a point.

Q.3 (a) Prove that $\sqrt{2}$ is an irrational number. [5]

(b) In usual notation, Prove that $E(x) = e^{x}; \forall x \in R$. [5]

OR

Q.3 (a) State and Prove the Archimedean property of R . [5]

(b) For all real numbers x and y ,prove that : (1) $|x+y| \le |x| + |y|$ [5]

(2) $|x\,y| = |x|\,|y|$.

Q.4 (a) Prove that C (x + y ) = C(x) C(y) - S(x) S(y). [5]

(b) Prove that a set is closed iff its complement is open. [5]

OR

Q.4 (a) State and Prove Bolzano-Weierstrass theorem for a set. [5]

(b) If S and T are sets of real numbers then Prove that the following

(i) $S \subset T \Longrightarrow S' \subset T'$ (ii) $(S \cup T)' = S' \cup T'$. [5]

Q.5 (a) Show that a continuous function on a closed interval [a,b] [5]

attains its bounds at least once in [a,b].

(b) Prove that limit of a function is unique if exists. [5]

OR

**Q.5 (a)** Prove that a function f : $[a,b] \to$R is continuous at a point c of [5]
$[a,b]$ iff $\lim_{n \to \infty} c_n = c \Rightarrow \lim_{n \to \infty} f(c_n) = f(c)$.

(b) Let f and g be two functions defined on some neighborhood [5]



(P.T.O)

of point a such that $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$, then

prove that $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{l}{m}, m \neq 0$.

Q.6 (a) State and prove Darboux's theorem for derivable function. [5]

(b) Prove that $\frac{x}{1+x} < \log(1+x) < x$, for all $x \geq 0$. [5]

OR

Q.6 (a) Define Uniformly Continuous function. [5]

Prove that the function $f(x) = x^2$ is uniformly continuous on [-1,1].

(b) Prove that a function which is derivable at a point is necessarily continuous at that point. Is the converse true? Justify your answer. [5]

—X—

(4)