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SARDAR PATEL UNIVERSITY

[86]

BSc Examination [Semester: V]

Subject: Physics Course: US05CPHY22

Mathematical Methods

Date: 26-12-2020, Saturday **Time: 02.00 pm to 04.00 pm** **Total Marks: 70**

INSTRUCTIONS:

1 Attempt all questions.
2 The symbols have their usual meaning.
3 Figures to the right indicate full marks.

Q-1 Multiple Choice Questions: [Attempt all] [10]

(i) The orthogonality condition for curvilinear co-ordinates is _____.

(a) $\frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial \vec{r}}{\partial v} = 0$ (b) $\frac{\partial \vec{r}}{\partial v} \cdot \frac{\partial \vec{r}}{\partial v} = 0$

(c) $\frac{\partial u}{\partial \vec{r}} \cdot \frac{\partial v}{\partial \vec{r}} = 0$ (d) $\frac{\partial \vec{r}}{\partial u} \cdot \frac{\partial u}{\partial v} = 0$

(ii) For curvilinear coordinates $\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} =$ ________.

(a) $\frac{h_1 h_3}{h_2} \frac{\partial \vec{r}}{\partial w}$ (b) $\frac{h_1 h_2}{h_3} \frac{\partial \vec{r}}{\partial w}$

(c) $\frac{h_1}{h_2 h_3} \frac{\partial \vec{r}}{\partial w}$ (d) $\frac{h_3 h_2}{h_1} \frac{\partial \vec{r}}{\partial w}$

(iii) For Hermite's function, $H_0(x) =$ ______.

(a) 0 (b) -1

(c) 1 (d) -4

(iv) For Legendre's equation, ______.

(a) $k = n \text{ or } k = -n - 1$ (b) $k = n \text{ or } k = -n$

(c) $k = 1 \text{ or } k = -1$ (d) $k = n \text{ or } k = n - 1$

(v) For Bessel's polynomial, the generating function is given by______.

(a) e^{2tx-t^2} (b) e^{2x-t^2}

(c) $e^{\frac{x}{2}(t^2-1)}$ (d) $e^{\frac{x}{2}(t-\frac{1}{t})}$

(vi) For a Fourier series of a periodic function $f(x)$ in $[-\pi, \pi]$, the coefficients $b_n =$_____.

(a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$ (b) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$

(c) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$ (d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$

(vii) In complex representation of a Fourier series, $\alpha_n =$ ________.

(a) $\frac{1}{\tau}\int_0^{\tau} f(t) \cos n\omega t\ dt$ (b) $\frac{2}{\tau}\int_0^{\tau} f(t) \cos n\omega t\ dt$

(c) $\frac{3}{\tau}\int_0^{\tau} f(t) \cos n\omega t\ dt$ (d) $\frac{4}{\tau}\int_0^{\tau} f(t) \cos n\omega t\ dt$

(viii) The problem of finding an equation of an approximating curve, which passes through as many points as possible is called ________.

(a) Curve fitting (b) Interpolation

(c) Telegraphy equation (d) Extrapolation

(ix) The forward difference operator Δ defined as ________.

(a) $\Delta y_i = y_{i-1} - y_i$ (b) $\Delta y_i = y_i - y_{i-1}$

(c) $\Delta y_i = y_{i+1} - y_i$ (d) $\Delta y_i = y_i - y_{i+1}$

(x) The backward difference operator ∇ defined as ________.

(a) $\nabla y_i = y_{i-1} - y_i$ (b) $\nabla y_i = y_i - y_{i-1}$

(c) $\nabla y_i = y_{i+1} - y_i$ (d) $\nabla y_i = y_i - y_{i+1}$

Q-2 State True or False. [Attempt all] [8]

(1) For curvilinear coordinates $ds^2 = h_1^2\, du^2 + h_2^2\, dv^2 + h_3^2\, dw^2$.

(2) For the cylindrical system the unit vectors are $\hat{e}_r, \hat{e}_\theta$ and $\hat{e}_\phi$.

(3) $J_n(\mu)$ is the coefficient of h^n in the expansion of $(1 - 2\mu h + h^2)^{-1/2}$.

(4) $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ is a Bessel's differential equation.

(5) The phase angle is given by $\phi_n = \tan^{-1}\left(\frac{\beta_n}{\alpha_n}\right)$.

(6) The sine series for $f(x)$ is given by $\frac{2}{\pi}\sum_{n=1}^{\infty} \sin nx \int_0^{\pi} f(\vartheta)\sin n\vartheta\, d\vartheta$ when $0 \ll x \ll \pi$.

(7) For a function $y = f(x)$, for a given table of values $(x_k, y_k), k = 1, 2, \ldots n$, the process of estimating the value of y, for any intermediate value of x is called interpolation.

(8) The shift operator E is defined as $Ef(x) = f(x+h)$.

Q-3 Answer the following questions in short. (Attempt any ten) [20]

(1) Define curvilinear coordinates.

(2) Write down Laplacian in terms of orthogonal curvilinear coordinates.

(3) If $u = x + 4,\ v = y - 2,\ w = 3z + 1$, show that u, v, w are orthogonal.

(4) Write Hermite's differential equation.

(5) Using equation: $H_n(x) = e^{x^2}(-1)^n \frac{d^n e^{-x^2}}{dx^n}$, find out $H_1(x)$.

(6) Show that $P_n(-\mu) = (-1)^n P_n(\mu)$.

(7) Write cosine series for $f(x)$ when $0 \le x \le \pi$. (Note: derivation is not required)

(8) Write one dimensional wave equation.

(9) Write telegraphy equation.

(10) Define interpolation.

(11) Derive an equivalent equation of a straight line for $y = ax^b$.

(12) For a shift operator E, show that $E = \Delta + 1$.

Q.4 Long Answer Questions. (Attempt any four) [32]

(1) Derive expression of gradient in terms of orthogonal curvilinear system.

(2) Derive expression of curl in terms of orthogonal curvilinear system.

(3) Derive the series solution of Legendre differential equation in the form of descending power of x.

(4) Derive the series solution of Bessel's differential equation in the form of ascending power of x.

(5) Write the Fourier series for a periodic function $f(x)$ defined in the interval $[-\pi, \pi]$. Derive the coefficients a_0, a_n and b_n of the series.

(6) Derive the Fourier series for a complex periodic function $f(t)$ defined in $(-\infty, \infty)$. Also find the coefficients α_n and β_n.

(7) Derive Lagrange's interpolation formula.

(8) Using the method of least squares, find the straight line $y = ax + b$ that fits the following data.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	15	17	19	14	10	7

Use the normal equations of least square fitting that fits a straight line $y = ax + b$:

$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i, \quad a\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$$

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