SEAT No.________ No. of Printed Pages : 3

[85]

SARDAR PATEL UNIVERSITY

BSc Examination [Semester: V] NC

Subject: Physics Course: US05CPHY02

Mathematical Physics

Date: 26-12-2020, Saturday **Time: 2.00 pm to 04.00 pm** **Total Marks: 70**

INSTRUCTIONS:

1. Attempt all questions.
2. The symbols have their usual meaning.
3. Figures to the right indicate full marks.

Q-1 Multiple Choice Questions: [Attempt all] **10**

(i) The matrix $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ is called _____.

(a) Symmetric matrix (b) Null matrix

(c) Asymmetric matrix (d) Unit matrix

(ii) The orthogonality condition for curvilinear co-ordinates is _____.

(a) $\frac{\partial r}{\partial u} \cdot \frac{\partial u}{\partial u} = 0$ (b) $\frac{\partial r}{\partial v} \cdot \frac{\partial r}{\partial v} = 0$

(c) $\frac{\partial u}{\partial r} \cdot \frac{\partial v}{\partial r} = 0$ (d) $\frac{\partial r}{\partial u} \cdot \frac{\partial r}{\partial v} = 0$

(iii) For a matrix $= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$, $|A| =$_____.

(a) 0 (b) 1

(c) -1 (d) -4

(iv) For Legendre's equation, _____.

(a) $k = 1 \; or \; k = -1$ (b) $k = n \; or \; k = -n$

(c) $k = n \; or \; k = -n - 1$ (d) $k = n \; or \; k = n - 1$

(v) The generating function for Bessel's polynomial is _____.

(a) e^{2tx-t^2} (b) e^{2x-t^2}

(c) $e^{\frac{x}{2}(t-\frac{1}{t})}$ (d) $e^{\frac{x}{2}(t-1)}$

(vi) The coefficients a_0 for Fourier series of a periodic function $f(x)$ in $[-\pi, \pi]$ =_____.

(a) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$ (b) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\, dx$

(c) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx$ (d) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$

(vii) In complex representation of a Fourier series, $\alpha_n =$ ________.

(a) $\frac{1}{\tau}\int_0^{\tau} f(t)\ \sin n\omega t\ dt$ (b) $\frac{1}{\tau}\int_0^{\tau} f(t)\ \cos n\omega t\ dt$

(c) $\frac{2}{\tau}\int_0^{\tau} f(t)\ \sin n\omega t\ dt$ (d) $\frac{2}{\tau}\int_0^{\tau} f(t)\ \cos n\omega t\ dt$

(viii) $\frac{\partial^2 u}{\partial x^2} = CL\frac{\partial^2 u}{\partial t^2} + (CR + GL)\frac{\partial u}{\partial t} + GRu$ is called the ________.

(a) Heat equation (b) Telegraphy equation

(c) Interpolation (d) Fourier series

(ix) "The best representative curve to the given set of observations is one for which E, the sum of the squares of the residuals, is minimum". This concept is known as ____.

(a) Principle of least squares (b) Principle of differentiation

(c) Principle of integration (d) None of these

(x) The backward difference operator ∇ defined as

(a) $\nabla y_i = y_{i-1} - y_i$ (b) $\nabla y_i = y_i - y_{i-1}$

(c) $\nabla y_i = y_{i+1} - y_i$ (d) $\nabla y_i = y_i - y_{i+1}$

Q-2 State True or False. [Attempt all] **8**

(1) For the cylindrical system these unit vectors are $\hat{e}_r, \hat{e}_\theta$ and $\hat{e}_z$.

(2) If $A^T = -A$, the matrix A is called unit matrix.

(3) $P_n(\mu)$ is the coefficient of h^n in the expansion of $(1 - 2\mu h + h^2)^{-1/2}$.

(4) $(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ is a Bessel's differential equation.

(5) The sine series for $f(x)$ is given by $\frac{2}{\pi}\sum_{n=1}^{\infty} \sin nx \int_0^{\pi} f(\vartheta) \sin n\vartheta\ d\vartheta$ when $0 \ll x \ll \pi$.

(6) The phase angle is given by $\phi_n = \tan^{-1}\left(\frac{\beta_n}{\alpha_n}\right)$.

(7) The problem of finding an equation of an approximating curve, which passes through as many points as possible is called curve fitting.

(8) For a function $y = f(x)$, for a given table of values $(x_k, y_k), k = 1, 2, \dots n$, the process of estimating the value of y, for any intermediate value of x is called extrapolation.

Q-3 Answer the following questions in short. (Attempt any ten) **20**

(1) Define (i) null matrix and (ii) unit matrix.

(2) Define curvilinear co-ordinates.

(3) If $u = x + 3,\ v = 2y - 1,\ w = 3z + 2$ show that u, v, w are orthogonal.

(4) Show that $P_n(-\mu) = (-1)^n P_n(\mu)$.

(5) Write Hermite's differential equation.

(6) Using equation: $H_n(x) = e^{x^2}(-1)^n \frac{d^n e^{-x^2}}{dx^n}$, find out $H_1(x)$.

(7) Write telegraphy equation.

(8) Write one dimensional wave equation.

(9) Write cosine series for $f(x)$ when $0 \le x \le \pi$. (Note: derivation is not required)

(10) Define interpolation.

(11) For a shift operator E, show that $\Delta = E - 1$.

(12) Derive an equivalent equation of a straight line for $y = ae^{bx}$.

Q.4 Long Answer Questions. (Attempt any four) 32

(1) Derive expression of gradient in terms of orthogonal curvilinear system.

(2) Discuss cylindrical co-ordinates as a special curvilinear system.

(3) Derive the series solution of Bessel's differential equation in the form of ascending power of x.

(4) Derive the series solution of Legendre differential equation in the form of descending power of x.

(5) Write the Fourier series for a periodic function $f(x)$ defined in the interval $[-\pi, \pi]$. Derive the coefficients a_0, a_n and b_n of the series.

(6) Derive the Fourier series for a complex periodic function $f(t)$ defined in $(-\infty, \infty)$. Also find the coefficients α_n and β_n.

(7) Derive Newton's forward difference interpolation formula.

(8) Derive Lagrange's interpolation formula.

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