SEAT No.________ **No. of Printed Pages : 2**

[111]

SARDAR PATEL UNIVERSITY

V. V. Nagar

B.Sc. Sem- V Examination

US05CMTH24 (Metric Spaces and Topological Spaces)

29th December 2020, Tuesday

02:00 pm to 04:00 pm

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The set of all cluster points of $A = \{1, \frac{1}{3}, \frac{1}{9}, \ldots, \frac{1}{3^n}, \ldots\}$ in $\mathbb{R}^1$ is...
(a) $\mathbb{N}$ (b) A (c) $A \cup \{0\}$ (d) $\{0\}$

(2) Let $(X, \mathcal{T})$ be a topological space and $Y \subset X$. Then Y is dense in X if
(a) $Y' = X$ (b) $Y' = \emptyset$ (c) $\overline{Y} = X$ (d) $\overline{Y} = \emptyset$

(3) Let ρ and σ be two metrics on M then which of the following is not a metric on M.
(a) $5\,\sigma$ (b) $\sigma - \rho$ (c) $\sigma + \rho$ (d) $3\,\sigma + 2\rho$

(4) Let $d : M \times M \to \mathbb{R}$ be a metric on M. Then which of the following is also a metric on M?
(a) $d_1(x, y) = \min\{1, d(x, y)\}$ (b) $d_1(x, y) = \max\{1, d(x, y)\}$
(c) $d_1(x, y) = \min\{0, d(x, y)\}$ (d) $d_1(x, y) = \max\{0, d(x, y)\}$

(5) Which of the following is not an open subset of $\mathbb{R}^1$.
(a) $(1,\ 3) \cup (5,\ 7)$ (b) $\mathbb{Q}$ (c) ϕ (d) $(-1,\ 2) \cup (0,\ 5)$

(6) In a topological space $(X, \mathcal{T})$, every $\mathcal{T}$-open set
(a) can not be a neighbourhood of all its points (b) is $\mathcal{T}$-closed also
(c) is a neighbourhood of all its points (d) none

(7) If $E = [1,\ 3) \cup \{4\} \subset \mathbb{R}^1$, then $\overline{E}\ldots$?
(a) $[1,\ 3) \cup \{4\}$ (b) $[1,\ 4]$ (c) $[1,\ 3] \cup \{4\}$ (d) $[1,\ 4)$

(8) Consider $M = [0, 1]$ with discrete metric. Find $B[1/4;\ 1/2] = \ldots$
(a) $(0,\ 1)$ (b) $[0,\ 1]$ (c) $\{1/4\}$ (d) $\mathbb{R}$

(9) Which of the following is not a closed subset of $\mathbb{R}$.
(a) $[3,\ 5] \cup [1,\ 7)$ (b) $\mathbb{R}$ (c) $\{1, 3, 5, 7, 9\}$ (d) None of these

(10) Let $X = \{a, b\}$. Then for which of the following $\mathcal{T}$, $(X, \mathcal{T})$ is not connected?
(a) $\{X, \emptyset, \{a\}\}$ (b) $\{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$
(c) $\{X, \emptyset, \{b\}\}$ (d) None of these

Q.2 Do as directed: [08]

(1) True or False: The closure of any subset of a metric space is always closed.

(2) True or False: Every function on Discrete matric space may not be continuous.

(3) True or False: Arbitrary intersection of open set is open set.

(4) True or False: In any metric space (M, ρ), M is always closed set.

(5) True or False: Every convergence sequence in a Metric space is a Cauchy sequence.

(6) Consider $\mathbb{R}$ with discrete metric. Then B[4; 0.99]=...

(7) Consider $\mathbb{R}$ with discrete metric. Then B[-5; 5]=...

(8) If $E = B[2;\ 5]$, then $\overline{E}$ in $\mathbb{R}^1$ is...

Q.3 Attempt any Ten. [20]

(1) Define Topological space and give its one example.
(2) Is $(0, 2]$ a $\mathcal{U}$-neighbourhood of 1? Justify!
(3) Are closed interval of $\mathbb{R}$, u- closed ? where u ia usual Topology for $\mathbb{R}$.
(4) Prove that $\{a\}$ is closed set in usual Topology.

(P.T.O.)

(5) Prove that every subset of $\mathbb{R}_d$ is open.
(6) Define continuity of a function.
(7) Show that if ρ is a metric for a set M, then so is 4ρ.
(8) If $\{x_n\}$ is a convergent sequence in $\mathbb{R}_d$, then show that there exist a positive integer N such that $x_N = x_{N+1} = x_{N+2} = \ldots$.
(9) Define: (i) Convergence of sequence in metric space (ii) Cauchy sequence.
(10) Let A be an open subset of the metric space M. If $B \subset A$ is open in A, then prove that B is open in M.
(11) Is arbitrary union of closed sets is closed? Justify!.
(12) Let f be a continuous real valued function on $[a, b]$, then prove that f is bounded.

Q.4 Attemt any Four. [32]

(a) Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Prove that A is $\mathcal{T}$-open set iff A contains a $\mathcal{T}$-neighbourhood of each of its points.

(b) If $\{G_\alpha : \alpha \in \Lambda\}$ is a collection of $\mathcal{U}$-open subsets of $\mathbb{R}$ then prove that $\cup\{G_\alpha : \alpha \in \Lambda\}$ is a $\mathcal{U}$-open set.

(c) Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ are topological spaces and f be a mapping of X into Y. Then f is continuous iff inverse image under f of every $\mathcal{T}_2$ closed set is $\mathcal{T}_1$ closed set.

(d) Show that every open subset G of $\mathbb{R}$ can be written $G = \cup I_n$, where $I_1,\ I_2,\ I_3, \ldots$ are a finite or a countable number of open intervals which are mutually disjoint.

(e) Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X and A' be the set of all cluster points of A. Prove that A is $\mathcal{T}$-closed iff $A' \subset A$.

(f) Let (M, d) be a metric space and $d_1(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$. Is d_1 a metric on M? Justify!

(g) Prove that if $(X, \mathcal{T})$ is disconnected iff there is a non empty proper subset of X that is both $\mathcal{T}-$ open and $\mathcal{T}-$ closed.

(h) Define interior of a set. Let $(X, \mathcal{T})$ be a topological space and A be a subset of X. Then prove That $Int(A)$ is the largest open subset of A.

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