SEAT No. ________ **No. of Printed Pages : 3**

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-V : Examinations : 2020-21

Subject : Mathematics US05CMTH22(T) Max. Marks : 70

Theory Of Real Functions

Date: 26/12/2020, Saturday Timing: 02.00 pm - 04.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] If $\lim_{x\to a} f(x)$ exists but $f(a)$ does not exist then f posseses a discontinuity of

[A] removable type [B] first type
[C] second type [D] first type from left

[2] If $f(x) = |x+2|$ then f is continuous from ____ at $x = -2$.
[A] both the sides [B] left only [C] right only [D] no sides

[3] $\lim_{x\to 0-} e^{\frac{1}{x}} =$
[A] 0 [B] 1 [C] ∞ [D] $-\infty$

[4] Maclaurin's theorem is a special case of _____ theorem.
[A] Rolle's [B] Lagrange's Mean Value
[C] Cauchy's Mean Value [D] Taylor's

[5] Which of the following functions does not satisfy atleast one condition of Lagrange's Mean Value theorem on $[-1,1]$?
[A] x^2 [B] $\sin x$ [C] $|x|$ [D] e^x

[6] Which of the following functions satisfy all the conditions of Lagrange's Mean Value theorem on $[-1,1]$?
[A] $-|x|$ [B] $|x|$ [C] $[x]$ [D] e^x

[7] If $\lim_{(x,y)\to(2,4)} f(x,y) = 3f(2,4)$ where, $f(2,4) \neq 0$ then
[A] $\lim_{(x,y)\to(2,4)} f(x,y)$ does not exist [B] f is continuous at $(6,12)$
[C] f is discontinuous at $(2,4)$ [D] f is continuous at $(4,2)$

[8] $\lim_{(x,y)\to(0,0)} \frac{x\sin(x^2+y^2)}{x^2+y^2} =$
[A] 0 [B] 1 [C] 2 [D] 3

[9] The necessary condition for a function f to have an extreme value at $(2,4)$ is
[A] $f_x(2,4) = 0,\ f_y(2,4) \neq 0$ [B] $f_x(2,4) \neq 0,\ f_y(2,4) = 0$
[C] $f_x(2,4) \neq 0,\ f_y(2,4) \neq 0$ [D] $f_x(2,4) = 0,\ f_y(2,4) = 0$

[10] If $f_{xx}(1,1) = R$, $f_{yy}(1,1) = S$ and $f_{xy}(1,1) = T$ then in which of the following case nothing can be concluded regarding extreme value of a function $f(x,y)$ at $(1,1)$?
[A] $RT - S^2 < 0$ [B] $RT - S^2 > 0$ [C] $RT - S^2 >= 0$ [D] $RT - S^2 = 0$

Q: 2. In the following, depending on the type of question either fill in the blank or answer whther a statement is true false **08**

[1] If $\lim_{x\to 0-} f(x) \neq \lim_{x\to 0+} f(x)$ then also $\lim_{x\to 0} f(x)$ can exist. (TRUE or FALSE?)

[2] Function $f(x) = |x|$, $\forall x \in R$ is discontinuous at 0.(TRUE or FALSE?)

[3] Rolles's theorem is applicable to the function $f(x) = x^2 - x$ on $[0,1]$ (TRUE or FALSE?)

[4] Function $f(x) = -x^3$ is decreasing on $[0,1]$. (TRUE or FALSE?)

[5] Fill in the blank. : $\lim_{y\to 1}\lim_{x\to 3} \frac{x+y}{x-y} =$ ____.

[6] Fill in the blank. : $\lim_{(x,y)\to(3,2)} (x^2 - xy) =$ ____.

[7] If $f(x,y) = x^2 + y^2$ then f has an extreme value at $(0,0)$ (TRUE or FALSE?)

[8] If $f(x,y) = x^4 + 4x^2y^2 + y^4$ then f has a maximum at $(0,0)$ (TRUE or FALSE?)

Q: 3. Answer ANY TEN of the following. **20**

[1] Is the function $f(x) = |x+1|, x \in R$ continuous at x = - 1? Justify.

[2] Examine the function $f(x) = \begin{cases} x^2 + 2x & \text{when } x \neq 3 \\ 15, & \text{when } x = 3 \end{cases}$ for continuity at $x = 3$

[3] Evaluate : $\lim_{x\to 0} \frac{\sqrt{4+x} - 2}{x}$.

[4] Explain the geometric meaning of Lagrange's Mean Value theorem

[5] Is Rolle's theorem applicable to $f(x) = 2x + 1$ on $[0,2]$? Why?

[6] In usual notations write the Lagrange's and Cauchy's forms of remainders of Maclaurin's expansion.

[7] Show that the following function is discontinuous at $(2,3)$

$$f(x,y) = \begin{cases} 2x + 3y^3 \ ; & \text{when } (x,y) \neq (2,3) \\ 0 \ ; & \text{when } (x,y) = (2,3) \end{cases}$$

[8] Evaluate : $\lim_{(x,y)\to(1,1)} \frac{4^{(x-y)} - 1}{x-y}$

[9] Evaluate : $\lim_{(x,y)\to(2,3)} \frac{\sin(3xy-18)}{\tan^{-1}(xy-6)}$

[10] Can a function $f(x,y) = x^2 + 5xy + y^2$ have an extreme value at $(1,1)$? Why?

[11] Show that $y^2 + x^2y + x^4$ has a minimum at $(0,0)$.

[12] State the necessary conditions for a function $z = f(x,y)$ to attain extreme values at a point (a,b)

Q: 4. Attempt ANY FOUR of the following questions. **32**

[1] Let f and g be two functions defined on some neighbourhood of a such that $\lim_{x\to a} f(x) = l$ and $\lim_{x\to a} g(x) = m$. Prove that $\lim_{x\to a} [f(x) + g(x)] = l + m$

[2] Show that a function $f : [a,b] \to \Re$ is continuous at point c of [a,b] iff

$$\lim_{n\to\infty} c_n = c \Longrightarrow \lim_{n\to\infty} f(c_n) = f(c)$$

[3] If $f'(c) > 0$, then prove that f is an increasing function at point $x = c$.

[4] Prove that, $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$.

[5] Show that $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ does not exist.

[6] State and prove a sufficient condition for a function $f(x,y)$ to be continuous at a point (a,b).

[7] Investigate the maxima and minima of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$

[8] Show that $f(xy, z - 2x) = 0$ satisfies, under suitable conditions, the equation $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$. What are these conditions?

——X——