

SEAT No. \_\_\_\_\_

No. of Printed Pages : 3

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Sardar Patel University, Vallabh Vidyanagar

B.Sc Sem: V Subject : Mathematics

Date: 24/12/2020 US05CMTH21 [Real Analysis] Time: 2.00 to 4.00 Max.Marks: 70

Q.1 Choose the correct option for each of the following.

[10]

(1) Which of the following is an ordered field ?

- (a)  $\mathbb{Q}$  (b)  $\mathbb{R}$  (c)  $\mathbb{N}$  (d) none of these

(2) The Smallest number of a set, if exists is ....

- (a) the supremum of the set (b) the infimum of the set (c) not unique (d) none

(3) The supremum of  $\left\{\frac{1}{m} + \frac{1}{n} / m, n \in \mathbb{N}\right\}$  is ....

- (a) 0 (b) 1 (c) 2 (d) none

(4) Every open interval in  $\mathbb{R}$  is ..... set

- (a) an open (b) a closed (c) open and closed (d) none

(5) The derived set of  $A = \{1, 2, 3, 4\}$  is ....

- (a)  $A$  (b)  $\mathbb{R}$  (c)  $\emptyset$  (d)  $\mathbb{Z}$

(6) The closure of  $\mathbb{Q}$ , i.e.  $\bar{\mathbb{Q}}$  is

- (a)  $\mathbb{N}$  (b)  $\mathbb{Q}$  (c)  $\emptyset$  (d)  $\mathbb{R}$

(7) The Range of sequence is always .....

- (a) empty (b) infinite (c) non-empty (d) none

(8) Every convergent sequence is

- (a) oscillating (b) bounded (c) unbounded (d) none

(9) A positive term series  $\sum \frac{1}{n^p}$  is convergent iff \_\_\_\_\_.

- (a)  $p = 1$  (b)  $0 < p < 1$  (c)  $p > 1$  (d)  $p < 0$

(10) A series  $\sum u_n$  is convergent then  $\lim_{n \rightarrow \infty} u_n$

- (a)  $\neq 0$  (b)  $= 0$  (c)  $= 1$  (d) does not exist

[13]

(P.T.O.)

Q.2 Do as directed.

[8]

- (1) The infimum of  $\left\{\frac{(-1)^n}{n} / n \in \mathbb{N}\right\}$  is ....
- (2) If  $S = \{0,5\} \cup \{5,6,7\}$  then the greatest element of  $S$  is .....
- (3) If  $S_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$ ,  $\forall n \in \mathbb{N}$  then  $\bigcap_{n=1}^{\infty} S_n = \dots\dots\dots$
- (4) ..... is a limit point of a set  $S = \left\{\frac{1}{n} / n \in \mathbb{N}\right\}$
- (5) The range of sequence  $\{(-1)^n / n \in \mathbb{N}\}$  is .....
- (6) True Or False: A sequence without limit point is bounded.
- (7) True Or False :If a positive term series  $\sum_{n=1}^{\infty} u_n$  is convergent then its partial sums is bounded above.
- (8) True Or False :The series  $\sum_{n=1}^{\infty} \frac{n}{n+1}$  is convergent.

Q.3 Attempt anyTEN:

[20]

- (1) Define : An Ordered Field.
- (2) Prove that the greatest lower bound of a set  $S$  is unique ,if it exist.
- (3) Find the g.l.b and l.u.b of  $\left\{1 + \frac{(-1)^n}{n} / n \in \mathbb{N}\right\}$  if they exist.
- (4) Prove that every open set is a union of open intervals.
- (5) Define: An open set.
- (6) Define: A limit point of a set .
- (7) Define: A convergent sequence .
- (8) Prove that every convergent sequence is bounded.
- (9) Prove that  $\lim_{n \rightarrow \infty} \frac{3+\sqrt{2}n}{n} = \sqrt{2}$
- (10) Define : Infinite series
- (11) Prove that the series  $\sum_{n=1}^{\infty} \frac{1}{n!}$  is convergent.
- (12) Investigate the behaviour of the series whose  $n^{th}$  term is  $n \frac{1}{n}$ .

[32]

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