SEAT No.________

[99]

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Sardar Patel University

B.Sc. Sem-V Mathematics

December 2020

US05CMTH05 – Number Theory

Date: 30-12-2020
Time: 02:00 P.M. to 04:00 P.M.

Total: 70 marks

Note:

(i) (a, b) is gcd of two numbers a and b.

(ii) $[a, b]$ is lcm of two numbers a and b.

Q-1 MCQs **[10]**

1. Which of the following is **not** true for all $a, b \in N$?
   A. $[a, 1] = 1$
   B. $[a, a] = a$
   C. $[a, b] = [b, a]$
   D. $\frac{ab}{(a,b)} = [a, b]$

2. What is decimal representation of binary number 101010?
   A. 40
   B. 42
   C. 41
   D. 43

3. If product of all positive divisors of a is denoted by $P(a)$ then, $P(23) =$ ____
   A. 1
   B. 7
   C. 23
   D. 46

4. If μ denotes the Mobious function, then $\mu(9) =$ ____
   A. 1
   B. -1
   C. 0
   D. 2

5. Which of the following is first Fermat's number?
   A. 1
   B. 5
   C. 17
   D. 27

6. If $[x]$ is the greatest integer function, then which of the following is **not** true?
   A. $[x] \le x$
   B. $[[x]] = [x]$

(P.T.O.)

C. $[1.5] = 2$ D. $x = [x] + a, 0 \leq a < 1$

7. $5^{2022} \equiv$ ____ $(mod\ 7)$
   A. 0 B. 1
   C. 5 D. 25

8. What is 5^{th} Fibonacci number?
   A. 21 B. 8
   C. 5 D. 34

9. $\phi(128) =$ ______
   A. 54 B. 500
   C. 32 D. 64

10. If $\frac{x}{2} \equiv 24(\text{mod } 3)$ then $x =$ ______
   A. 1 B. 3
   C. 2 D. 0

Q-2 Do as directed **[08]**

1. True or False: $c|b$ & $b|a \Rightarrow a|c$.
2. Suppose there exists $x, y \in Z$ such that $ax + by = 1$ then $(a, b) =$ ____
3. True or False: If $a \equiv b(\text{mod } n)$ then $n|(a - b)$.
4. A positive integer a is said to be a perfect number if $S(a) =$ ____.
5. True or False: If $(x, y) = 1$ then the equation $xy = z^n$ has no solution.
6. Any relation is called equivalent, if it is reflexive, symmetric and _____.
7. True or False: If $(a, b) = 1$ then $\phi(ab) = \phi(a)\phi(b)$.
8. If $(a, m) = 1$ then $a^{\phi(m)} \equiv$ ____$(\text{mod } m)$.

Q-3 Attempt any Ten short questions. **[20]**

1. Prove that $(ac, bc) = (a, b)|c|, \forall c \in Z$.
2. Find $(1420, 230)$.
3. Define: Greatest Common Divisor (GCD)
4. If $a_1 \equiv a_2(\text{mod } n)$ then prove that $a_1 + c \equiv a_2 + c(\text{mod } n)$ for $c \in Z$.
5. If a is prime number, then what is the formula for sum of all positive divisors of a?

6. Define: Mersenne number.
7. What is the condition for a number divisible by 3?
8. Find number of multipliers of 7 among integers from 200 to 500.
9. Define: Reduced Residue System modulo m.
10. Define equivalent relation.
11. Define: Order of an element.
12. State the Chinese remainder theorem.

Q-4 Attempt any Four out of Eight. [32]

1. Let g be a positive integer greater than 1. Prove that every positive integer a can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + \cdots + c_1 g^1 + c_0$ where, $n \geq 0, c_i \in Z, 0 \leq c_i < g, c_n \neq 0$.
2. State and prove fundamental theorem of arithmetic.
3. Prove that any prime factor of M_p is greater than p(p is prime).
4. Prove that $F_0 F_1 F_2 \cdots F_{n-1} = F_n - 2$. Where F_n is n^{th} Fermat's number.
5. Find a necessary and sufficient condition that a positive integer is divisible by 11.
6. Prove that positive integer solution of $x^{-1} + y^{-1} = z^{-1}$, $(x, y, z) = 1$ is of the form $x = a(a + b), y = b(a + b), z = ab$ where $a, b > 0$, $(a, b) = 1$.
7. Solve the following system of congruent equations $x \equiv 1(mod\ 4), x \equiv 3(mod\ 5), x \equiv 2(mod\ 7)$.
8. Prove that the sum of $\phi(m)$ positive integers less than $m(m > 1)$ and relatively prime to m is $\frac{m}{2}\phi(m)$.

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