SEAT No.________ No. of Printed Pages : 2

[110]

SARDAR PATEL UNIVERSITY

B.Sc.SEM-V EXAMINATION (NC)

29^{th} December 2020, Tuesday

02.00 p.m. to 04.00 p.m.

US05CMTH04 (Abstract Algebra-I) Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) In Klein 4-group $G = \{e, a, b, c\}$, $b^2 = \ldots$

(a) e (b) b (c) c (d) a

(2) A permutation σ is said to be odd permutation if signature of σ is ....

(a) 2 (b) -1 (c) 1 (d) -2

(3) ____________ is generator of group ${Z_5}^*$

(a) $\bar{3}$ (b) $\bar{1}$ (c) $\bar{4}$ (d) $\bar{5}$

(4) External direct sum of Z_2 is ....

(a) Klein 4- group (b) Q (c) Z (d) Z_2

(5) Every infinite cyclic group has exactly .... generators .

(a) 3 (b) 1 (c) 2 (d) 4

(6) Centre of $\mathbb{Z}$ is ....

(a) $\mathbb{Z}$ (b) 2 (c) N (d) 1

(7) Every noncyclic group of order 4 is isomorphic to ____

(a) Klein 4-group (b) Z (c) N (d) Z_4

(8) $O(-i)$ in $\{\pm 1, \pm i\}$ is ....

(a) 3 (b) 4 (c) 1 (d) 2

(9) Every group of order ______ is abelian group .

(a) 2 (b) 5 (c) 4 (d) 6

(10) Multiplicative inverse of 5 in ${Z_7}^*$ is ....

(a) 3 (b) 6 (c) 2 (d) 1

Q.2 Do as directed: [08]

(1) True or False: The identity of group is unique.

(2) True or False: The intersection of two subgroups of a group G may not be a subgroup of G.

(3) True or False: The product of permutation of 4 symbols commutative.

(4) True or False: Let H be any subgroup of group G. Then $aH = H \Leftrightarrow a \in H$.

(5) Multiplicative inverse of ~~5~~ 6 in ${Z_7}^*$ is ....

(6) ______ is generator of group Z_n.

(7) Order of S_4 is ....

(8) Signature of every transposition is ....

 (P.T.O.)

Q.3 Attempt the short questions. **[Any Ten]** [20]

(1) Prove that every subgroup of Z is of the form nZ,for some $n \in Z$.
(2) Prove that $(M_2(R), +)$ is a group.
(3) Prove that intersection of two subgroups of a group G is also a subgroup of G.
(4) Find $O(2)$ in Z.
(5) State and prove left cancelation law in group G.
(6) Let H be any subgroup of group G. Then prove that $aH = H \Leftrightarrow a \in H$.
(7) Prove that $\theta : Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z.
(8) Express the inverse of cycle (12453) as a product of transpositions.
(9) Define Group and finite group.
(10) Is the product of permutation of 4 symbols commutative? Justify.
(11) Prove that S_n is a finite group of order $n!$.
(12) Prove that $Z(G)$,the centre of group G is a subgroup of G.

Q.4 Attempt the Long questions. **[Any Four]** [32]

(a) Let H and K be subgroups of group G.Then prove that HK is subgroup of G iff $HK = KH$.

(b) Let H and K be finite subgroups of group G such that HK is a subgroup of G. Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$.

(c) Let G be a finite cyclic group of order n ,then prove that G has $\phi(n)$ generators.

(d) If G is cyclic group of order n and $a^m = e$ for some $m \in \mathbb{Z}$ then prove that n/m.

(e) Let $G = (a)$ be a finite cyclic group of order n. Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n.

(f) State and Prove First isomorphism theorem.

(g) Prove that S_n is non commutative group of order $n!$.

(h) State and prove Cayley's theorem.

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