SEAT No. ____________  No. of Printed Pages : 3

[43] # Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-V : Examinations : 2020-21 [NC]

Subject : Mathematics  US05CMTH02  Max. Marks : 70

Real Analysis-II

Date: 26/12/2020, Saturday  Timing: 02.00 pm - 04.00pm

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Instruction : The symbols used in the paper have their usual meaning, unless specified.

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Q: 1. Answer the following by choosing correct answers from given choices. [10]

[1] Every convergent sequence is
[A] oscillating  [B] bounded  [C] unbounded  [D] none

[2] The sequence $\{2^n\}$
[A] is convergent  [B] diverges to $\infty$  [C] diverges to $-\infty$  [D] oscillates finitely

[3] If a sequence $\{S_n\}$ is such that $\lim\limits_{n\longrightarrow\infty}\frac{S_{n+1}}{S_n}=l$ then $\lim\limits_{n\longrightarrow\infty}S_n=\infty$ if
[A] $l<1$  [B] $l\leqslant 1$  [C] $l>1$  [D] $l\geqslant 1$

[4] If $\lim\limits_{n\to\infty}u_n=1$, for a positive term series $\sum\limits_{n=1}^{\infty}u_n$ then the series
[A] converges to 0  [B] converges to 1  [C] converges to 2  [D] cannot converge

[5] The series $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is
[A] bounded  [B] convergent  [C] divergent  [D] none

[6] If $\sum\limits_{i=1}^{\infty}u_i$ is a positive term series and $\sum\limits_{i=1}^{n}u_i<100, \forall n$ then the series
[A] is convergent  [B] diverges to $+\infty$  [C] diverges to $-\infty$  [D] none

[7] $\lim\limits_{(x,y)\to(6,\pi)} x^2\tan\frac{y}{x}=$
[A] 36  [B] $36\sqrt{3}$  [C] $12\sqrt{3}$  [D] $3\sqrt{12}$

[8] If $f(x,y)=x^3y^3-3x^2y^2$ then $f_y(0,1)=$
[A] 0  [B] 1  [C] 2  [D] 3

[9] For a sufficiently many times differentiable function $f(x,y)$ its Taylor's expansion about $(-1,2)$ is a series in powers of
[A] $x+1$ and $y-2$  [B] $x-1$ and $y+2$
[C] $x-1$ and $y-2$  [D] $x+1$ and $y+2$

[10] For a function $f$, if $f_x(a,b)<f_y(a,b)$ then at $(a,b)$, $f$ has
[A] no extreme value  [B] a minimum  [C] a maximum  [D] a stationery point

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false [08]

[1] Sequence $\{-(0.2)^n\}$ is convergent. (True or False?)

[2] A sequence may have more than one limits. (True or False?)

[3] For a sequence $\sum_{n=1}^{\infty} S_n$, if $\lim_{n\to\infty} S_n = 7$ then the series is not convergent. (True or False?)

[4] The serues $2 + 4 + 8 + \cdots + 2^n + \ldots$ is convergent. (True or False?)

[5] Fill in the blank. : $\lim_{(x,y)\to(-1,-2)} (3x^2 + 5y^2) =$ _____

[6] $\lim_{x\to 1}\lim_{y\to -1} \dfrac{x^2-y^2}{x^2+y^2} \neq \lim_{y\to -1}\lim_{x\to 1} \dfrac{x^2-y^2}{x^2+y^2}$ (True or False?)

[7] If $f''(1) = 0$ then $f$ has an extreme value at 1. (True or False?)

[8] $f(x) = 100x - 99, x \in R$ has no extreme value.(True or False?)

Q: 3. Answer ANY TEN of the following. [20]

[1] Define : Limit Superior and Limit Inferior

[2] Prove that every convergent sequence is bounded.

[3] Show that $\underset{n\longrightarrow\infty}{lim} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$

[4] Test $\sum_{n=1}^{\infty} \dfrac{n^2-1}{3n^2}$ for convergence

[5] Test $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ for convergence

[6] Why the series $\sum_{n=0}^{\infty} \dfrac{5n^3+4n}{n^3}$ cannot converge?

[7] Define : Limit of a function

[8] Evaluate : $\lim_{(x,y)\to(1,1)} \dfrac{e^{(x-y)}-1}{x-y}$

[9] Evaluate : $\lim_{(x,y)\to(2,3)} \frac{\sin(3xy-18)}{\tan^{-1}(xy-6)}$

[10] Can a function $f(x, y) = x^3 - xy - y$ have an extreme value at $(-1, -1)$? Why?

[11] Does the function $f(x, y) = x^100 + y^100$ defined on $R^2$, have minimum at $(0, 0)$? Why?

[12] State Maclaurin's theorem

Q: 4. Attempt ANY FOUR of the following questions. [32]

[1] State and prove the Bolzano-Weierstarss theorem for sequence

[2] Show that the sequence $\{r^n\}$ converges iff $-1 < r \leqslant 1$.

[3] Prove that the series $\sum \frac{1}{n}$ does not converge.

[4] State and prove the comparision test of first type.

[5] If $f$ and $g$ are two functions defined on same neighbourhood of a point $(a, b)$ such that $\lim_{(x,y)\to(a,b)} f(x, y) = l$, and $\lim_{(x,y)\to(a,b)} g(x, y) = m$ then prove that

$$\lim_{(x,y)\to(a,b)} (f.g) = \lim_{(x,y)\to(a,b)} f. \lim_{(x,y)\to(a,b)} g = l.m$$

[6] If $f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & \text{if } x^2 + y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$ then find $\lim_{(x,y)\to(0,0)} f(x, y)$

[7] State and prove Taylor's theorem

[8] Investigate the maxima and minima of the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$

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