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SARDAR PATEL UNIVERSITY

B.Sc. SEM- V EXAMINATION (NC)

24th December 2020 ,Thursday

2.00 p.m to 4.00 p.m

Sub.: Mathematics (US05CMTH01)

(Real Analysis-I)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The derived set of a set is....
(a) open and closed set (b) not open set (c) closed set (d) open but not closed

(2) The supremum of the set $\{1+(-1)^n : n \in N\}$ is
(a) 0 (b) 1 (c) 2 (d) not exists

(3) The set $\{1, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{3}, ...\}$ is...
(a) open and closed set (b) open set (c) closed set (d) neither open nor closed

(4) The field which does not have the least upper bound property is ...
(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ (d) $\mathbb{R}$

(5) Every uniformly continuous function is.....
(a) continuous (b) not continuous (c) unbounded (d) none

(6) A function f is said to have a dicontinuity of the first kind from right at $x = c$ if...
(a) $\lim_{x \to c^-} f(x) \neq f(c)$ (b) $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$ exists but not equal
(c) $\lim_{x \to c^+} f(x) \neq f(c)$ (d) None

(7) The Set $\bigcup(\frac{1}{n}, 1-\frac{1}{n})$ is
(a) $(0,1)$ (b) $\{0,1\}$ (c) $[0,1]$ (d) $[0,1)$

(8) The continuous function on closed interval is.....
(a) not bounded (b) open set (c) bounded (d) none.

(9) If $f'(c) < 0$, then function f is..... at c.
(a) not derivable (b) increasing (c) descreasing (d) none of these

(10) In $(0, \pi/2)$, the function $C(x)$ is
(a) strictly decreasing (b) strictly increasing (c) stationary (d) none

Q.2 Do as directed: [08]

(1) True or False: A finite set may have limit point.

(2) True or False: Every differentiable function is continus.

(3) True or False: Every infinite subset of real numbers has Gratest and Smallest member.

(4) True or False: The derived set of a set is closed.
(5) True or False: A set which is not closed is always open set.
(6) The infimum of a set $\{-1+\frac{(-1)^n}{n} : n \in N\}$ is ....
(7) The infimum of a set $S = (2,6) \cup \{1,2,3,4,5,6,7,8\} = ...$
(8) The set $\{1+\frac{(-1)^n}{n} : n \in N\}$ has ... number of limit points.

Q.3 Attempt **any ten** in short: [20]

(1) Is every continuous function derivable? Justify your answer.
(2) Find the supremum and the infimum of a set $\{2+\frac{(-1)^n}{n} : n \in N\}$.
(3) Prove that supremum of a set S of numbers, if it exists, is unique.
(4) Let $m = inf(S)$, where S is a bounded set. Show that $m \in$ closure of S .
(5) Prove that $L(ab) = L(a) + L(b)$.
(6) Prove that the superset of a neighbourhood(nbd) of a point x is also a nbd of x.
(7) If f is derivable at point c then, show that function $1/f$ derivable at point c, where $f(c) \neq 0$.
(8) In usual notations, prove that $\lim_{x\to a}(f-g)(x) = l - m$.
(9) Prove that limit of a function is unique, if exists.
(10) Is $(S \bigcap T)' = S' \bigcap T'$? Justify your answer.
(11) Prove that $a^n = a.a.a.....a$ (n times).
(12) Show that the supremum of a bounded non-empty set S, when not a member of S, is a limit point of S.

Q.4 Attempt **any Four** : [32]

(a) Prove that the set of rational numbers is not order complete.
(b) In usual notations, prove that $E(x) = e^x$ for all $x \in R$.
(c) State and Prove Bolzano-Weierstrass theorem for a set.
(d) Prove that there exists a positive number π such that $C(\pi/2) = 0$ and $C(x) > 0$ for $0 \leq x < \pi/2$.
(e) In usual notations, prove that $\lim_{x\to a}\left(\frac{f}{g}\right)(x) = \frac{l}{m}$, provided $m \neq 0$.
(f) Show that a function $f : [a,b] \to \mathbf{R}$ is continuous at point c of $[a,b]$ iff $\lim_{n\to\infty} c_n = c \Rightarrow \lim_{n\to\infty} f(c_n) = f(c)$.
(g) Show that $log(1+x)$ lies between $\frac{x}{1+x}$ and x for all $x > 0$.
(h) State and prove Darboux's theorem for derivable function.

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