[21] No. of printed pages: 3

SARDAR PATEL UNIVERSITY
BSc (V Sem.) Examination
Friday, 22 November 2013
10.30 am – 1.30 pm
US05CMTH05 – Mathematics
Number Theory

Total Marks: 70

Note: Figures to the right indicates full marks.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

(1) (4676, 366) = ________.
(a) 6 (b) 2 (c) 1 (d) 4

(2) [12, 30] = ________.
(a) 60 (b) 30 (c) 6 (d) 360

(3) If 'a' is square number, then S(a) is ________.
(a) Even (b) 0
(c) Odd (d) Prime

(4) Fermat Number F_3=________.
(a) 3 (b) 13 (c) 65537 (d) 257

(5) T(810) = ________.
(a) 41 (b) 20 (c) 38 (d) 28

(6) ax+by=c has integer solution if and only if ________.
(a) (a, b) = a (b) (a, b) = b
(c) $(a,b)/c$ (d) $c/(a,b)$

(7) (x, y, z) = ________ is one of the relative prime solution of $x^2 + y^2 = z^2$ with 0<z<30.
(a) (5, 12, 13) (b) (20, 21, 29)
(c) (2, 5, 7) (d) (5, 13, 17)

(8) $ca \equiv cb(\bmod \cdot n) \Rightarrow a \equiv b(\bmod \cdot n)$ only if ________.
(a) (c, b) = 1 (b) (c, a) = b
(c) (c, a) = 1 (d) (c, n) = 1

(9) Reduced residue system modulo m contains ________ elements.
(a) m (b) $\phi(m-1)$ (c) $\phi(m)$ (d) $\phi(m+1)$

(10) $\phi(1008)$=________.
(a) 288 (b) 1007 (c) 144 (d) 126

Q.2 Answer the following in short. **(Attempt Any Ten)** [20]

(1) Prove that $[a,b,c] = \dfrac{abc}{(ab,bc,ca)}, \forall a,b,c>0$.

(2) Find g.c.d of two numbers by using Euclidean algorithm.

(3) Prove that $(a+b)[a,b]=b[a, a+b], \ \forall a,b>0$.

(4) If 'a' is not square number but odd integer then prove that S(a) is even integer.

(5) Prove that two successive Fibonacci numbers are relatively prime.

(6) If x is any real number and n is a positive integer then prove that $\left[\frac{[x]}{n}\right]=\left[\frac{x}{n}\right]$

(7) If $a_1 \equiv b_1 \pmod{n}$ & $a_2 \equiv b_2 \pmod{n}$ then prove that $a_1a_2 \equiv b_1b_2 \pmod{n}$.

(8) If $a_1 \equiv b_1 \pmod{n}$ then prove that $a^m \equiv b^m \pmod{n}, \ \forall m \in N$ by using mathematical induction method.

(9) If $a \equiv b \pmod{m}, a \equiv b \pmod{n}$ and $K = [m,n]$ then prove that $a \equiv b \pmod{K}$

(10) Is {27, 80, 96, 113, 64} a C. R. S. modulo 5? Justify.

(11) If $a^{m-1} \equiv 1 \pmod{m}; (a,m)=1 \,\&\, a^n \not\equiv 1 \pmod{m}$ for any proper divisor n of m-1 then prove that m is prime.

(12) Prove that $\phi(p^K) = p^K\left(1-\frac{1}{p}\right)$ where p is prime.

Q.3

(a) State and prove Fundamental theorem of divisibility. [05]

(b) Prove that $(a^{m}-1, a^{n}-1) = a^{(m,n)}-1$. [05]

OR

Q.3

(a) State and prove Unique Factorization Theorem. [05]

(b) Let g be a positive integer greater than 1 then prove that every positive integer 'a' can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + \ldots + c_1 g + c_0$ where $n \geq 0, \ ci \in Z, 0 \leq ci < g, c_n \neq 0$. [05]

Q.4

(a) Prove that odd prime factor of M_p (p>2) has the form 2pt+1 for some integer t. [05]

(b) If a & b are relatively prime number then prove that [05]

(i) $T(ab) = T(a) \cdot T(b)$

(ii) $S(ab) = S(a) \cdot S(b)$

(iii) $P(ab) = P(a)^{T(b)} \cdot P(b)^{T(a)}$

OR

Q.4

(a) Prove that $S(a) < a\sqrt{a}, \forall a > 2$. [05]

(b) Prove that odd prime factor of $a^{2^n} + (a > 1)$ is of the form $2^{n+1} + 1$ for some $t \in Z$. [05]

Q.5

(a) Prove that the integer solution of $x^2 + 2y^2 = z^2; (x, y) = 1$ can be expressed as $x = \pm(a^2 - 2b^2); y = 2ab; z = a^2 + 2b^2$. [05]

(b) Solve the equation: 7x+19y=213. [05]

OR

Q.5

(a) Prove that the general integer solution of $x^2+y^2=z^2$ with x, y, z > 0; (x, y) = 1 and y is even is given by $x = a^2 - b^2$; $y = 2ab$; $z = a^2 + b^2$ where a, b > 0; (a, b) = 1 and one of a, b is odd and the other is even. [05]

(b) State and prove necessary and sufficient condition that a positive integer is divisible by 11. Is 527590 divisible by 11? [05]

Q.6 Show that Euler's function is multiplicative and hence find ϕ (142296). [10]

OR

Q.6 State and prove Chinese Remainder theorem and hence solve the system of congruences: $x \equiv 2 \pmod 3; x \equiv 3 \pmod 5; x \equiv 2 \pmod 7$. [10]

* * *