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SARDAR PATEL UNIVERSITY
BSc (V Sem.) Examination
2013
Wednesday, 20th November
10.30 am - 1.30 pm
US05CMTH04 - Abstract Algebra I

Total Marks: 70

Note: Figures to the right indicate marks.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

(1) $(Z_7^*, \bullet)$ is a group then Z(G) = __________.

(a) ϕ (b) $\{\bar{1}\}$ (c) $\{\bar{1},\bar{3},\bar{5}\}$ (d) G

(2) O (i) in (C^*, •) is __________.
(a) 1 (b) 3 (c) 4 (d) 2

(3) Multiplicative inverse of 6 in Z_7^* is __________.

(a) 3 (b) 6 (c) 2 (d) 1

(4) A cyclic group of order __________ is not a simple.
(a) 15 (b) 17 (c) 19 (d) 23

(5) __________ is a generator of group Z_5^*.

(a) $\bar{0}$ (b) $\bar{1}$ (c) $\bar{2}$ (d) $\bar{4}$

(6) Every cyclic group of order b4 is isomorphic to __________.
(a) Klein's 4 Group (b) Z (c) N (d) Z_4

(7) If $\theta: G \to G'$ is homomorphism, then θ is one-one iff ker θ =__________.
(a) ϕ (b) {e} (c) $\{\theta\}$ (d) none

(8) If H = Z_2, K = Z_2 where $Z_2 = \{\bar{0},\bar{1}\}$ then H$\oplus$K is __________.
(a) summation (b) Z_4 (c) Klein's 4 Group (d) none

(9) O (S_n/A_n) = __________.
(a) 1 (b) 2 (c) 4 (d) 8

(10) A permutation σ is said to be odd permutation of signature of σ is __________.
(a) -1 (b) +1 (c) 0 (d) alternate

Q.2 Answer the following in short. **(Attempt Any Ten)** [20]

(1) Prove that every group has unique identity element.
(2) Prove or disprove: Union of two subgroups of a group is also a subgroup.
(3) State Cancellation laws for group.
(4) Find all generators of group $\{\pm 1, \pm i\}$ if possible.
(5) If G is a cyclic group, then prove that G is an Abelian group.
(6) Let H be any subgroup of group G, then prove that aH = H $\Leftrightarrow$ a $\in$ H.
(7) Prove that isomorphic image of an Abelian group is also Abelian.
(8) Let $\theta: G \to G'$ be a homomorphism, prove that ker. θ is a subgroup of G.

(9) Prove that $\theta: Z \to Z$ defined by $\theta(n) = -n$ is an automorphism of Z.

(10) Let G = $\langle a \rangle$ be any cyclic group of order 6. H = {e, a^2, a^4}, K = {e, a^3}. Show that G is an internal direct product of H and K.

(11) Prove that mapping $\varepsilon: S_n \to \{+1, -1\}$ given by $\sigma \to \varepsilon\sigma$ is a homomorphism of S_n onto the multiplicative group {-1, 1}.

(12) Prove that composition of two permutations need not be commutative.

Q.3

(a) Prove that G is abelian iff G = Z(G). [05]

(b) Let H and K be two subgroups of G, then prove that HK is a subgroup of G iff HK = KH. [05]

OR

Q.3

(a) Prove that (G, $\bullet$) is a non-commutative group, where G is set of all 2×2 non-singular matrices. [05]

(b) Prove that intersection of any number of subgroup of a group G is also a subgroup of G. [05]

Q.4 Let G be any cyclic group and H is a subgroup of G, then prove that H is cyclic. [10]

OR

Q.4 State and prove Lagrange's theorem an Euler's theorem. [10]

Q.5

(a) State and prove first isomorphism theorem. [05]

(b) Prove that every infinite cyclic group has only one non-trivial automorphism. [05]

OR

Q.5

(a) State and prove third isomorphism theorem. [05]

(b) Let G = $\langle a \rangle$ be any finite cyclic group of order n. Then prove that the mapping $\theta: G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n. [05]

Q.6

(a) Prove that external direct product of two groups forms a group. [05]

(b) Prove that A_n, the set of all even permutations of S_n, is a normal subgroup of S_n and $O(A_n) = \frac{n!}{2}$ [05]

OR

Q.6

(a) Prove that the set S_n of all permutations on n symbols forms a non-commutative group. [05]

(b) Prove that the external direct product of two cyclic groups each of order 2 is the Klein's 4 group. [05]

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