CA

[31] No. of printed pages : 3

SARDAR PATEL UNIVERSITY
BSc (V Sem.) Examination
2013
Monday, 18th November
10.30 am - 1.30 pm
US05CMTH03 - Metric Spaces

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Answer the following by selecting correct choice from the given options. [10]

(1) If sequence $\{S_n\}_{n=1}^{\infty}$ is a sequence ℓ^2 in then __________.

(a) $\sum_{n=1}^{\infty} S_n = \infty$ (b) $\sum_{n=1}^{\infty} S_n \leq \infty$ (c) $\sum_{n=1}^{\infty} S_n < \infty$ (d) none

(2) The set of all cluster points of a set (1, 2) is __________.

(a) ϕ (b) [1, 2] (c) (1, 2) (d) R

(3) The convergent sequence in a metric space M can not converge to __________.

(a) two limit points (b) two distinct limit points
(c) unique limit point (d) none of these

(4) __________ subset of R_d is always open.

(a) only some (b) only one (c) no (d) every

(5) In usual notation $\overline{Q}$ = __________.

(a) R (b) ϕ (c) Q (d) Q′

(6) Any _________ subset of metric space is always closed.

(a) finite & infinite (b) infinite (c) finite (d) none

(7) __________ metric space is complete.

(a) R_d (b) R^2 (c) ℓ^{∞} (d) none of above

(8) Metric space __________ is compact.

(a) [1, 3] (b) [2, 4) (c) (0, 5] (d) (3, 6)

(9) __________ image of compact metric space is compact.

(a) Any (b) Bounded (c) Continuous (d) None

(10) Continuous function on a compact metric space is __________.

(a) unbounded (b) bounded
(c) discontinuous (d) none

Q.2 Answer the following in short. **(Attempt Any Ten)** [20]

(1) Define: Open Ball.

(2) For M= [0, 1] with usual metric, find open ball of radius $\frac{1}{2}$ about $\frac{1}{4}$.

(3) Define Cauchy Sequence in a metric space.

(4) In metric space $\langle M, \rho \rangle$, prove that M is open set.

(5) Prove that any singleton set in R_d is open.

(6) In usual notations prove that $\mathbf{E} \subset \overline{\mathbf{E}}$.

(7) State Heine Borel property.

(8) If A = (5, 7) and ρ is absolute vale metric then find diam. (A).

(9) Define: Connected Set.

(10) Show that the range of a continuous function on a compact metric space is bounded.

(11) Define: Uniform Continuous function.

(12) If the real valued function f is continuous on [a, b] then prove that f is uniformly continuous.

Q.3 Let $\langle M,d \rangle$ be a metric space and let $d_1(x, y) = \frac{d(x,y)}{1+d(x,y)}$ then prove that d_1 is a metric on M. [10]

OR

Q.3 Prove that convergent sequence of points in a metric space $\langle M,\rho \rangle$ is Cauchy. Is the converse true? Justify. [10]

Q.4

(a) If E is a subset of a metric space M, then prove that $\overline{E}$ is closed in M. [05]

(b) If F_1 and F_2 are closed subset of metric space M, then prove that $F_1 \cup F_2$ is closed in M. [05]

OR

Q.4

(a) Show that every open subset G of R' can be written as G-U In where I_1, I_2, ... are finite or countable number of open intervals which are mutually disjoint. [05]

(b) Let $\langle M_1,\rho_1 \rangle$ and $\langle M_2,\rho_2 \rangle$ be metric spaces. Let $f: M_1 \to M_2$ then prove that f is continuous on M_1 iff $f^{-1}(G)$ is open in M_1 whenever G is open in M_2. [05]

Q.5

(a) State and prove Nested Interval theorem. [05]

(b) If $\langle M,\rho \rangle$ is complete metric space and A is closed subset of M, then prove that $\langle M,\rho \rangle$ is also complete. [05]

OR

Q.5

(a) State and prove Picard's fixed point theorem. [05]

(b) Show that the subset A of R is connected iff whenever $a \in A, b \in B$ with a<b, then $c \in A$ for every c such that $a < c < b$. [05]

Q.6

(a) Let f be continuous function from the compact metric space M_1 into the metric space M_2, then prove that the range $f(M_1)$ is also compact. [05]

(b) Let $\langle M_1, \rho_1 \rangle$ be a compact metric space. If f is continuous function form M_1 into a metric space $\langle M_2, \rho_2 \rangle$, then prove that f is uniformly continuous. [05]

OR

Q.6 Let $\langle M_1, \rho_1 \rangle$ be metric space and let A be a dense subset of M_1. If f is a uniformly continuous function from $\langle A_1, \rho_1 \rangle$ into a complete metric space $\langle M_2, \rho_2 \rangle$ then prove that f can be extended to a uniformly continuous function F from M_1 into M_2. [10]

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