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SARDAR PATEL UNIVERSITY
BSc (V Sem.) Examination
2013
Friday, 15th November
10.30 am - 1.30 pm
US05CMTH02 - Real Analysis 2

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Answer the following by selecting correct choice from the given options. [10]

(1) Sequence $\{(-1)^n\}$ ____________.
(a) is convergent (b) is divergent
(c) oscillates finitely (d) oscillates infinitely

(2) Every convergent sequence is ____________.
(a) oscillating (b) bounded (c) unbounded (d) none

(3) Sequence $\{-2^n\}$ ____________.
(a) converges (b) oscillates infinitely
(c) diverges to $+\infty$ (d) diverges to $-\infty$

(4) If a sequence $\{s_n\}$ is a sequence of partial sums of the series, then S_3=____________.
(a) $u_1+u_2+u_3$ (b) $u_1 \times u_2 \times u_3$
(c) $\frac{u_1+u_2+u_3}{3}$ (d) $u_1-u_2-u_3$

(5) A positive term series $\sum \frac{1}{n^p}$ is convergent iff ____________.
(a) p=1 (b) p<1 (c) p>1 (d) p<0

(6) A positive term series $\sum_{n=1}^{\infty} u_n$ diverges if $\lim_{n\to\infty} (u_n)^{1/n}$ ____________.
(a) ≤ 1 (b) > 1 (c) $= 1$ (d) none

(7) $\lim_{x\to 1} \lim_{y\to -1} \frac{4x^3y^2}{x^2+y^2}$ = ____________.
(a) 3 (b) 0 (c) 1 (d) 2

(8) If $f(x,y) = x^3y^3-3x^2y^2$ then $f_y(0, 1)$ = ____________.
(a) 0 (b) 1 (c) 2 (d) 3

(9) The extreme value of f(a,b) is called maximum if f(x,y) - f(a,b) is ____________.
(a) alternate +ve & -ve (b) positive
(c) negative (d) none

(10) A stationary point is called saddle point of function f if it is ____________ point.
(a) extreme (b) not an extreme
(c) stationary (d) none

Q.2 Answer the following in short. **(Attempt Any Ten)** [20]

(1) Check the convergence of the sequence $\{1+(-1)^n\}$.

(2) If $\{a_n\}$ and $\{b_n\}$ are two sequences then show that $\lim_{n \to \infty}(a_n+b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$.

(3) Show that $\lim_{n \to \infty} \frac{(3n+1)(n-2)}{n(n+3)} = 3$.

(4) Investigate the behaviour of the series whose n^{th} term is $\sin\frac{1}{n}$.

(5) State D' Alembert's ratio test.

(6) If $\sum u_n = n$ and $\sum v_n = v$ then prove that $\sum(u_n+v_n) = u+v$.

(7) If $f(x, y) = 2x^2-xy+2y^2$ then find fx and fy at the point (1, 2).

(8) Evaluate: $\lim_{(x,y)\to(0,0)} \frac{\sin^{-1}(xy-2)}{\tan^{-1}(3xy-6)}$.

(9) Define: Repeated limits.

(10) Derive Maclaurin's theorem by using Taylor's expansion.

(11) Write down the Taylor's expansion of $f(x, y)$ about the point (a, b).

(12) For $f(x, y) = y^2+x^2y+x^4$, find $f_{xx}.f_{yy}-(fxy)^2$ at (0, 0).

Q.3

(a) State and prove Bolzano-Weierstrass theorem for sequence. [05]

(b) Show that $\lim_{n \to \infty} \sqrt[n]{n} = 1$. [05]

OR

Q.3

(a) Show that $\{r^n\}$ converges iff $-1<r\leq 1$. [05]

(b) State and prove Cauchy's principle of convergence. [05]

Q.4

(a) State and prove comparison test in limit form. [05]

(b) Show that the series $\frac{1}{(\log 2)^p}+\frac{1}{(\log 3)^p}+\ldots+\frac{1}{(\log n)^p}\ldots$ diverges for p > 0.

OR

Q.4

(a) State and prove Cauchy's root test. [05]

(b) Test the convergence of the series [05]

(i) whose n^{th} term is $(n^3+1)^{1/3} - n$

(ii) whose n^{th} term is $\frac{1}{n^{1+\frac{1}{n}}}$

Q.5

(a) If u = F (x, y, z) and z = (x, y) then find a formula for $\frac{\partial^2 u}{\partial x^2}$ in terms of the derivative of F and the derivative of z. [05]

(b) Show that $\frac{\partial^2 \theta}{\partial x \partial y} = -\frac{\cos 2\theta}{r^2}$ [05]

OR

Q.5

(a) Show that for given function limit exists at the origin but the repeated limit does not. [05]

$$f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) + y\sin\left(\frac{1}{x}\right) & ,xy \neq 0 \\ 0 & ,xy = 0 \end{cases}$$

(b) If V is a function of two variables x & y and $x = r\cos\theta,\ y = r\sin\theta$ then prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r}\frac{\partial v}{\partial r}$ [05]

Q.6 State and prove Taylor's theorem. Also expand $e^x \tan^{-1} y$ about (1, 1) up to second degree in powers of (x-1) and (y-1). [10]

OR

Q.6 State necessary and sufficient condition for f(x, y) to have an extreme value at (a, b). Also find maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$. [10]

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