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SARDAR PATEL UNIVERSITY
BSc (V Sem.) Examination
Saturday, 1st December 2012
2.30 - 5.30 pm
US05CMTH05 : Mathematics (Number Theory)

Total Marks: 70

Note: Figures to the right indicate full marks.

Q.1 Answer the following questions by selecting the most appropriate option. Write down the correct option in your answer book. [10]

(1) If K is any positive integer, then K^2+K+1 is _______ number.
(a) Prime (b) Square
(c) Not a Square (d) Even

(2) For integer a and b; a=bq+r; o≤r<b; then (a,b)=_______.
(a) 1 (b) (b, r)
(c) (a, r) (d) $|a|$

(3) If b is multiple of 'a', then (a, b)=_______.
(a) 1 (b) b
(c) a (d) $|a|$

(4) P(20)=________.
(a) 800 (b) 400
(c) 600 (d) 1000

(5) (u_{16}, u_{12})=_________.
(a) 4 (b) 3
(c) 5 (d) 2

(6) _________ is Mersenne number.
(a) 31 (b) 8
(c) 10 (d) 15

(7) An even and an odd integer are ______ for modulo 2.
(a) Congruent (b) May or May not Congruent
(c) Incongruent (d) Non of these

(8) The necessary and sufficient condition that $a \equiv b(\mathrm{mod}.m)$ is ______.
(a) $\frac{m}{a}$ (b) $\frac{m}{(a-b)}$
(c) $\frac{m}{b}$ (d) $\frac{(a-b)}{m}$

(9) For an integer m, $\phi(m)$________$m-1$, $\forall m>1$
(a) = (b) >
(c) ≥ (d) ≤

(10) If (m, n)= _______, then $\phi(m,n) \neq \phi(m)+\phi(n)$
(a) (3, 6) (b) (2, 2)
(c) (3, 4) (d) (4, 3)

Q.2 Write down the answer of **Any Ten** in short. [20]

(1) If K>0 is a common multiple of a and b, then prove that $\left(\frac{K}{a}, \frac{K}{b}\right)=\frac{K}{[a, b]}$

(2) If 'a' is a composite integer and q is its least positive divisor then prove that $q \le \sqrt{a}$.

(3) If n is an odd integer and n=ab, then prove that n can be decomposed as a difference of two square number.

(4) If 'x' is any real number and 'n' is a positive integer, then prove that $\left[\frac{[x]}{n}\right]=\left[\frac{x}{n}\right]$

(5) Find number of multiples of 7 among the integers from 200 to 500.

(6) Prove that $u_{n+3} = 3u_{n+1}-u_{n-1}$

(7) If $a \equiv b(\bmod m)$; $a \equiv b(\bmod n)$ and $K=[m,n]$ then show that $a \equiv b(\bmod K)$

(8) Find the positive integer solution of $x^2+ xy-6 =0$

(9) If $a_1 \equiv b_1(\bmod n)$ then prove that $ca_1 \equiv cb_1(\bmod n)$; $\forall\ C \in Z$

(10) Find order of 2 modulo 7.

(11) Check whether {26, 37, 48, 59, 10} is complete residue system modulo 5 or not.

(12) If (a, p)=1; p is prime, then prove that $a^{p-1} \equiv 1 \pmod p$

Q.3

(a) State and prove unique factorization theorem. [05]

(b) In usual notations prove that, $[a, b]\cdot(a, b)=ab$ [05]

OR

Q.3

(a) Let g be a positive integer greater than 1 then prove that every positive integer a can be written uniquely in the form $a=C_n g^n + C_{n-1} g^{n-1} + ... + C_1 g + C_0$ where $n \ge 0,\ C_i \in Z,\ 0 \le c_i < g,\ C_n \ne 0$ [05]

(b) State and prove fundamental theorem of divisibility. [05]

Q.4

(a) Define: Mobious Function with appropriate illustrations. Prove that Mobious Function is multiplicative. [05]

(b) Define: Mersenne Number with appropriate illustrations. Prove that any prime factor of Mp is greater then p. [05]

OR

Q.4

(a) Prove that $s(a)<a\sqrt{a},\ \forall a > 2$ [05]

(b) In usual notations prove that, $P(n!)=\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+....+\left[\frac{n}{p^{n+1}}\right]$ [05]

where, $p^m \le n < p^{m+1}$

Q.5

(a) Prove that the general integer solution of $x^2+y^2=z^2$ with x, y, z >0; (x, y) =1 and y is even is given by $x=a^2-b^2$, y=2ab, $z=a^2+b^2$; where a, b>0; (a, b)=1 and one of a, b is odd and the other is even. [05]

(b) Solve the equation: 7x+19y=213 [05]

OR

Q.5

(a) State and prove necessary and sufficient condition that a positive integer is divisible by 11. Is 765432 divisible by 11? Justify. [05]

(b) Solve the equation: 19x+20y=1909 [05]

Q.6 Define: Euler's Function. Prove that Euler's function is multiplicative. [10]

OR

Q.6 State and prove Chinese Remainder theorem and hence solve the system of congruences: $x\equiv1 \pmod 4$, $x\equiv3 \pmod 5$, $x\equiv2 \pmod 7$ [10]

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