[89] No. of printed pages: 03

SARDAR PATEL UNIVERSITY
B.Sc.. (Vth Semester) EXAMINATION
2012
Tuesday, 27th November
2.30 pm to 51.30 pm

US05CMTH02 – MATHEMATICS REAL ANALYSIS - 2

Total Marks : 70

Q.1 Answer the following question by selecting the most appropriate option. Write down the option in your answer book. [10]

1. If the sequence {Sn} is bounded then it __________
   (a) Oscillates infinitely (b) Oscillates finitely
   (c) Diverges to + ∞ (d) Diverges to - ∞
2. A number ξ is said to be a limit point of a sequence {Sn} if every neighbourhood of ξ contains _______ of members of the sequence.
   (a) Infinite Number (b) Finite Number
   (c) Alteast one Number (d) None of these
3. A sequence {Sn} is strictly increasing if for all n, ______
   (a) $S_{n+1} \geq S_n$ (b) $S_{n+1} \leq S_n$
   (c) $S_{n+1} < S_n$ (d) $S_{n+1} > S_n$.
4. For infinite series Σu_n if $\lim_{n \to \infty} u_n=0$, then the series ________
   (a) Converges always (b) Does not converge
   (c) Converges sometimes (d) None of these
5. The sequence of partial sums of a series with negative terms converges iff the sequence of partial sums is _______
   (a) Bounded Above (b) Unbounded Above
   (c) Unbounded Below (d) Bounded Below
6. The positive term geometric series $1 + r + r^2 + ...$ converges for __
   (a) $r < 1$ (b) $r > 1$ (c) $r = 1$ (d) None of these
7. A function is said to be continuous in a region if it is continuous at _____ of the given region.
   (a) Only one Point (b) Every Point
   (c) Some Point (d) Nowhere
8. A sufficient condition that a function is continuous in a closed region is that both the partial derivative exists and are _____ through out the region.
   (a) Equal (b) Unbounded (c) Bounded (d) None of these
9. The extreme value of f (a, b) is called maximum if sign of f(x,y) - f(a,b) is _______
   (a) Positive (b) Negative
   (c) Alternate +ve & -ve (d) None of these
10. A necessary condition for f(x,y) to have an extreme value at (a,b) is

that ________

(a) fx(a,b) = 0 (b) fy(a,b) = 0

(c) fxy(a,b) = 0 (d) fx(a,b)= 0 = fy(a,b)

Q.2 Write down the answer of **Any Ten** questions in short. [20]

1. Prove that every convergent sequence is bounded.
2. Define: Convergence of sequence. Check whether {Sn} = $\{\frac{1}{n}\}$ is convergent or not.
3. Show that $\{1+(-1)^n\}$ oscillates finitely.
4. Check the convergence of the series. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+$.........
5. Show that the series. $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+$......... is convergent.
6. Test the convergence of the serious $\sum \frac{1}{n^{1+\frac{1}{n}}}$
7. Evaluate $\lim_{(x,y)\to(0,0)} x \cdot \frac{\sin(x^2+y^2)}{(x^2+y^2)}$
8. Define continuity at a point for the function of two variables.
9. Define: limit of a function of two variables.
10. State Maclaurin's Expansion.
11. Define : Extreme Value.
12. Write down rules to find exterme value.

Q.3 (a) State and prove Bolzano-Weierstrass theorem for sequence. [05]

(b) Prove that a sequence can not converge to more than one limit. [05]

OR

Q.3 (a) State and prove Cauchy's first theorem on limits. [05]

(b) Prove that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded. [03]

Q.4 (a) State and prove comparison test of second order. [05]

(b) Prove that the positive term geometric series $1 + r + r^2 + ...$ converges for $r < 1$ and diverges to ∞ for $r \geq 1$. [05]

OR

Q.4 (a) State and prove Cauchy's Root Test. [05]

(b) Check the convergence of the series. [05]

$$\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + ...$$

Q.5 (a) Show that f(xy, z-2x) =0 satisfies, under suitable conditions, the [06] equation. $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial x} = 2x$. What are these conditions ?

(b) Show that the function. [05]

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at the origin.

OR

Q.5 (a) For the function $f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) + y\sin\left(\frac{1}{x}\right) & , xy \neq 0 \\ 0 & , xy = 0 \end{cases}$ [05]

Prove that limit exists at the origin but the repeated limits do not.

(b) If $f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2} & , \text{if } x^2 + y^2 \neq 0 \\ 0 & , \text{if } x = y = 0 \end{cases}$ [05]

then discuss about $\lim_{(x,y)\to(0,0)} f(x,y)$

Q.6 State and prove Taylor's theorem. Also, expand $f(x,y) = x^2y + 3y-2$ in powers of (x-1) and (y+2) [10]

OR

Q.6 Prove that the first four terms of the Maclaurin's expansion of

e^{ax} cosby are $1+ax+\frac{a^2x^2-b^2y^2}{2!} + \frac{a^3x^3-3ab^2xy^2}{3!}$

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