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SEAT No.________

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[64 & A-47]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - V) EXAMINATION
Thursday, 12-04-2018
MATHEMATICS : US05CMTH04
(Abstract Algebra - I)

Time : 02:00 p.m. to 05:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) .... is called trivial subgroup of group G .

(a) G (b) $\{e\}$ (c) $\{e, G\}$ (d) $\{0\}$

(2) Total .... binary operations can be defined on a set with two elements .

(a) 16 (b) 4 (c) 2 (d) 6

(3) Let R^* be the set of all nonzero real numbers and operation $*$ is defined as $a * b = \frac{1}{2}ab$. In group $(R^*, *)$ $a^{-1} =$ ....

(a) $a/4$ (b) $2/a$ (c) $4/a$ (d) a

(4) Every group of order .... is cyclic.

(a) 4 (b) 6 (c) 12 (d) 7

(5) Let $H = 4Z$ $G = Z$ then $H - 1 =$ ....

(a) $H + 1$ (b) $H + 3$ (c) $H - 3$ (d) $H + 4$

(6) If G is cyclic group of order n and $a^m = e$, for some $m \in Z$ then ....

(a) m/n (b) n/m (c) $m = n$ (d) $m = 0$

(7) If $G = R$, $G' = \{z \in C/|z| = 1 \}$ .Define $f : G \to G'$ by $f(a) = e^{2\pi ai}$ then $f(a+b)=$ ....

(a) $f(a) + f(b)$ (b) $f(a) - f(b)$ (c) $f(ab)$ (d) $f(a)f(b)$

(8) If $f : R \to R^+$ defined by $f(x) = 2^x$ then f is ....

(a) not one-one (b) not onto (c) onto (d) not homomorphism

(9) Order of S_4 is ....

(a) 3 (b) 12 (c) 24 (d) 4

(10) Ker $\varepsilon =$ ....

(a) A_n (b) e (c) ± 1 (d) S_n

(P.T.O.)

Que.2 Answer the following (Any Ten) 20

(1) Prove that a non-empty subset H of a group G is subgroup if $ab^{-1} \in H \;\forall\; a,b \in H$.

(2) Prove that intersection of two subgroups of a group G is also a subgroup of G.

(3) For group G , prove that $(a^{-1})^{-1} = a \;\forall\; a \in G$.

(4) If G is cyclic group of order n and $a^m = e$ for some $m \in \mathbb{Z}$ then prove that n/m .

(5) Give an example of finite cyclic group.Verify it.

(6) If G is a finite group and H a subgroup of G,then prove that $O(G) = O(H)(G:H)$.

(7) Prove that commutative subgroup G' of a group G is normal in G .

(8) Let $\theta : G \to G'$ be a homomorphism.Then prove that $Ker\theta$ is a normal subgroup of G.

(9) Prove that the mapping $\theta : Z \to Z_n$ defined by $\theta(a) = \bar{a}$ is a group homomorphism.Is it one one ?

(10) Let $G = H \times K$ be a direct product of H and K ,then prove that the mapping $p_K : G \to K$defined by $p_K(h,k) = k$ is group homomorphism with kernel $H' = \{(h, e_K)/h \in H\}$.

(11) Let$G = \{e,a,b,c\}$ be the Klein 4-group,$H = \{e,a\}$, $K = \{e,b\}$.Show that $G = H \times K$.

(12) Prove that every permutation can be written as a product of disjoint cycles.

Que.3 (a) Let H and K be finite subgroups of group G such that HK is a subgroup of G.Then prove that $O(HK) = \dfrac{O(H)O(K)}{O(H \cap K)}$. 5

(b) Let H and K be subgroups of group G.Then prove that HK is subgroup of G iff $HK = KH$. 5

OR

Que.3 (a) Prove that $(G,*)$ is a commutative group , where G is a set of all subsets of $\mathbb{R}$ and $*$ defined as $A * B = (A - B) \cup (B - A)$, $\forall A, B \in G$. 5

(b) Prove that intersection of two subgroups of a group G is also a subgroup of G . Is union of two subgroups of group a subgroup. ? Verify it . 5

Que.4 (a) Prove that every subgroup of an infinite cyclic group is also an infinite cyclic group. 5

(b) Let G be a finite cyclic group of order n . Then prove that G has unique subgroup of order d for every divisor d of n . 5

OR

Que.4 (a) State and prove Lagranges theorem , Euler's theorem and Fermat's theorem. 6

(b) Let G be a group and $a, b \in G$ such that $ab = ba$. If $O(a) = n$, $O(b) = m$ with m , n relatively prime,then prove that $O(ab) = mn$. 4

Que.5 (a) State and prove third isomorphism theorem. 5

(b) Let $G = \langle a \rangle$ be a finite cyclic group of order n.Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n. 5

OR

Que.5 (a) Prove that a subgroup H is normal in group G iff $xH = Hx \; \forall \; x \in G$. 3

(b) Let $G = \mathbb{R}$, $G' = \{z/z \in C \; , \; |z| = 1\}$,then prove that $G/\mathbb{Z} \simeq G'$. 3

(c) Let $G' = \{1, \rho, \rho^2, ..., \rho^{n-1}\}$ be the multiplicative group of n^{th} root of unity, where $\rho = e^{2\pi i/n}$.Then prove that $Z_n \simeq G'$. 4

Que.6 (a) State and prove Cayley's theorem for group . 5

(b) Prove that G is direct product of subgroups H and K iff
(i) every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$
(ii) $hk = kh$,$\forall$ $h \in H$, $k \in K$. 5

OR

Que.6 (a) Prove that $\sigma \in S_n$ can be expressed as a product of transpositions . Also prove that the number of transpositions in the decomposition of σ is either always odd or always even. 4

(b) Prove that the external direct product of two cyclic groups each of order 2 is the Klein 4-group. 4

(c) Prove that S_n/A_n is a cyclic group of order 2. 2

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