SEAT No.________ **No. of Printed Pages : 2**

[80]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER-V)EXAMINATION-2018
April 11, 2018, Wednesday
2.00 p.m. to 5.00 p.m.
US05CMTH03(MATHEMATICS)(Metric Spaces)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The set of all cluster points of $A = \{1, \frac{1}{3}, \frac{1}{9}, \ldots, \frac{1}{3^n}, \ldots\}$ in $\mathbb{R}^1$ is...
(a) $\mathbb{N}$ (b) A (c) $A \cup \{0\}$ (d) $\{0\}$

(2) Let ρ and σ be two metrics on M then which of the following is not a metric on M.
(a) $5\,\sigma$ (b) $\sigma - \rho$ (c) $\sigma + \rho$ (d) $3\,\sigma + 2\rho$

(3) Which of the following is not an open subset of $\mathbb{R}^1$.
(a) $(0,\ 4) \cup (5,\ 6)$ (b) $\mathbb{Q}$ (c) ϕ (d) $(-1,\ 3) \cup (0,\ 6)$.

(4) Consider $\mathbb{R}$ with usual metric. Then $B[7; 9]=\ldots$
(a) $(2,\ 16)$ (b) $(7,\ 9)$ (c) $(-2,\ 16)$ (d) $[2,\ 16]$.

(5) Which of the following subset of $\mathbb{R}^1$ is not connected?
(a) $\mathbb{R}$ (b) $\mathbb{Q}$ (c) $(4, 7)$ (d) $(1, 5) \cup (3, 8)$

(6) Let $A = [0,\ 2] \subset \mathbb{R}^1$. Which of the following subset of A is not an open subset of A?
(a) $[1,\ 2)$ (b) $[0,\ 2]$ (c) $(1,\ 2)$ (d) $(0,\ 2]$

(7) Which of the following subset of $\mathbb{R}_d$ is totally bounded?
(a) $[-2,\ 5]$ (b) $(3,\ 7)$ (c) $\{1, 2, ..., 3^{10}\}$ (d) $\mathbb{N}$

(8) Which of the following subset of $\mathbb{R}^1$ is not complete?
(a) $[2,\ 3]$ (b) $(\infty,\ 2]$ (c) $\{1, 2, ...n\}$ (d) $(0,\ 3]$

(9) In usual metric, which of the following is not a compact set?
(a) $[1,\ 3]$ (b) $\{1, 2, ..., 9^{20}\}$ (c) $\{0, 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{n}, ...\}$ (d) $[1,\ 3)$

(10) Let $f : [0, \frac{1}{3}] \to [0, \frac{1}{3}]$ be defined by $f(x) = x^2$, then which of the following is not true?
(a) f is a contraction. (b) Range of f is compact
(c) f is not uniformly continuous (d) Range of f is connected.

Q.2 Attempt any Ten: [20]

(1) Show that if ρ is a metric for a set M, then so is 2ρ.
(2) Define: (i) Convergence of sequence (ii) Cauchy sequence.
(3) Let (M_1, ρ_1) and (M_2, ρ_2) be two metric spaces and let $f : M_1 \to M_2$ be defined by $f(x) = 3$, $\forall x \in M_1$. Then show that f is continuous on M_1.
(4) Prove that every subset of $\mathbb{R}_d$ is open.
(5) Is the intersection of an infinite number of open sets is open? Justify!
(6) Let f be a continuous function from a metric space M_1 onto a metric space M_2. If M_1 is connected, then M_2 is also connected.
(7) Define: (i) Totally bounded set (ii) Complete metric Space.
(8) If (M, ρ) is a complete metric space and A is closed subset of M, Then prove that (A, ρ) is also complete.
(9) Prove that a bounded subset A of $\mathbb{R}^1$ is totally bounded.
(10) Prove that every contraction mapping is continuous.
(11) Define: (i) Compact metric space (ii) Finite intersection property.
(12) Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x$ is uniformly continuous.

(P.T.O.)

Q.3

(a) Let (M, d) be a metric space and let $d_1(x,y) = \dfrac{d(x,y)}{1+d(x,y)}$. Then show that d_1 is a metric on M. [5]

(b) The real valued function f is continuous at $a \in \mathbb{R}^1$ if an only if whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers converging to a, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ converges to $f(a)$. [5]

OR

Q.3

(c) Let (M, ρ) be a metric space. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points of M, then $\{s_n\}_{n=1}^{\infty}$ is Cauchy. Is converse true? Justify! [4]

(d) Let (M, ρ) be a metric space and $a \in M$. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$, then show that $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{L}{N}$, if $N \neq 0$. [6]

Q.4

(a) Every open subset G of $\mathbb{R}^1$ can be written $G = \bigcup I_n$, where $I_1, I_2, I_3, \ldots$ are a finite number or a countable number of open intervals which are mutually disjoint.(i.e. $I_m \cap I_n = \phi$ if $m \neq n$) [5]

(b) If E is any subset of the metric space M, Then show that $\overline{E}$ is closed. [5]

OR

Q.4

(c) Let M be a metric space. Then M is connected iff every continuous characteristic function on M is constant. [5]

(d) Let (M, ρ) be a metric space and let A be a proper subset of M. Then the subset G_A of A is an open subset of (A, ρ) iff there exist an open subset G_M of (M, ρ) such that $G_A = A \cap G_M$. [5]

Q.5

(a) If (M, ρ) is a complete metric space and A is closed subset of M, then prove that (A, ρ) is also complete. [4]

(b) State and prove Picard's fixed point theorem. [6]

OR

Q.5

(c) Let (M, ρ) be a metric space. The subset A of M is totally bounded iff every sequence of points of A contains a Cauchy subsequence. [5]

(d) State and prove generalized nested interval theorem. [5]

Q.6

(a) If M is a compact metric space, then prove that M has the Heine-Borel property. [6]

(b) Let $(M_1 \rho_1)$ be a compact metric space. If f is continuous function from M_1 into a metric space (M_2, ρ_2), then f is uniformly continuous on M_1. [4]

OR

Q.6

(c) The metric space M is compact iff whenever $\mathcal{F}$ is a family of closed subsets of M with the finite intersection property, then $\bigcap_{F \in \mathcal{F}} F \neq \phi$. [6]

(d) If the real valued function f is continuous on the compact metric space M, then f attains a maximum value at some point of M. Also, f attains a minimum value at some point of M. [4]

——X——