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SEAT No._____ No. of Printed Pages : 3

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# SARDAR PATEL UNIVERSITY

B.SC SEM-V EXAMINATION

09th April 2018, Monday

02.00 p.m. to 05.00 p.m.

US05CMTH01

(Real Analysis-I)

Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) The derived set of a set is....
(a) open and closed set (b) not open set (c) closed set (d) open but not closed.

(2) A function $f$ is said to have a removable discontinuity at $x = c$ if...
(a) $\emptyset$ (b) $limf(x) \neq f(c)$ (c) $limf(x) = f(c)$ (d) None.

(3) The supremum of the set $\{1 + (-1)^n : n \in N\}$ is
(a) 0 (b) 2 (c) 1 (d) not exists .

(4) If $A = \{1, 2, \ldots 100\}$, then the interior of $A$ is
(a) $\{0\}$ (b) $\phi$ (c) $A$ (d) not exists .

(5) The derivative of $E(x)$ is .....
(a) 1 (b) $E(x)$ (c) $x$ (d) 0.

(6) The infinite intersection of open set is.....
(a) open and closed set (b) not open set (c) closed set (d)all of these.

(7) The continuous function on closed interval is.....
(a) not bounded (b) open set (c) bounded (d) none.

(8) In $(0, \pi/2)$, the function $C(x)$ is
(a) strictly decreasing (b) strictly increasing (c) stationary (d) none.

(9) If $f'(c) > 0$, then function $f$ is..... at $c$.
(a) not derivable (b) increasing (c) descreasing (d) none of these.

(10) The function $|x|$ is.....at origin.
(a) not derivable (b) derivable (c) not continuous (d) none of these.

Q.2 Attempt **any ten** in short: [20]

(1) Find the supremum and the infimum of a set $\{1 + \frac{(-1)^n}{n} : n \in N\}$.

(2) Prove that $|x| = max(x, -x)$.

(3) Define complete ordered field.

(4) Define limit point of set with example.

(5) Show that the supremum of a bounded non-empty set $S$, when not a member of $S$, is a limit point of $S$.

(6) Define: Continuity and Uniform continuity of a function

C.P.T.O.

(7) Evaluate: $\lim_{x \to 0} \frac{\sqrt{(4+x)}-2}{x}$.

(8) Is the function $f(x) = |x|$ derivable at origin ?

(9) Prove that the superset of a neighbourhood(nbd) of a point $x$ is also a nbd of $x$.

(10) Is every continuous function derivable? Justify your answer.

(11) Let $M = sup(S)$, where $S$ is a bounded set. Show that $M \in$ closure of $S$ .

(12) Define: Closed and Open sets.

Q.3(a) Prove that the set of rational numbers is not order complete. [6]

(b) For all real numbers x and y, show that: [4]

(i) $|x - y| \geq ||x| - |y||$,

(ii) $|xy| = |x||y|$ .

OR

Q.3(c) In usual notations, prove that $E(x) = e^x$ for all $x \in R$. [6]

(d) Show that there is no rational number whose square is 7. [4]

Q.4(a) State and Prove Bolzano-Weierstrass theorem for a set. [6]

(b) In usual notations, Prove that $(S \cup T)' = S' \cup T'$. [4]

OR

Q.4(c) Prove that there exists a positive number $\pi$ such that $C(\pi/2) = 0$ and $C(x) > 0$ for $0 \leq x < \pi/2$. [6]

(d) Show that the derived set of a set is closed. [4]

Q.5(a) In usual notations, prove that $\lim_{x \to a}(\frac{f}{g})(x) = \frac{l}{m}$, provided $m \neq 0$. [6]

(b) If f is continuous on $[a, b]$ then prove that it attains its bounds at least once in $[a, b]$. [4]

OR

Q.5(c) Examine the function for continuity at $x = 0, 1$ and 2. Also discuss the kind of discontinuity where $f(x)$ defined on R by [6]

$$f(x) = \begin{cases} -x^2, & \text{if } x \leq 0 \\ 5x - 4, & \text{if } 0 < x \leq 1 \\ 4x^2 - 3x, & \text{if } 1 < x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$$

(d) Prove that limit of a function is unique, if exists. [4]

Q.6(a) If $f'(c) > 0$ , then prove that $f$ is monotonic increasing function at point $x = c$. [5]

(b) State and prove Darboux's theorem for derivable function. [5]

OR

Q.6(c) Show that $log(1+x)$ lies between $\frac{x}{1+x}$ and $x$ for all $x > 0$. [5]

(d) Define uniform continuity. Prove that the function $x^2$ is uniformly continuous on $[-1, 1]$. [5]