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(A-3) Seat No: ____________ No of printed pages : 3

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER V) EXAMINATION (NC)
Monday, 16th May, 2016
MATHEMATICS : US05CMTH06
(MECHANICS - 1)

Time : 10.30 a.m. to 01.30 p.m. Maximum Marks:70

Que.1 Answer the following. 10

(1) [Angular Momentum] = ...

(a) ML^2T^{-2} (b) MLT^{-1} (c) ML^2T (d) ML^2T^{-1}

(2) 1 ft = ...cms .

(a) 30 (b) 30.38 (c) 30.48 (d) 12

(3) 1 Poundal = ... ounces .

(a) 16 (b) 26 (c) 36 (d) 15

(4) The point of concurrence of the altitudes of a triangle is called the .... of a triangle .

(a) Centroid (b) Circum center (c) Orthocenter (d) Incenter

(5) If we push a body by a rod then the force exerted is called ....

(a) tension (b) moment (c) weight (d) thrust

(6) Moment of vector (X , Y , Z) about the perpendicular to the plane O_{xy} at the point (b , a) is M = ...

(a) $(x-b)Y-(y-a)X$ (b) $(x+a)Y+(y+b)X$ (c) $(x-a)Y-(y-b)X$

(d) $(x+a)Y-(y+b)X$

(7) A branch of mechanics which deals with the motion of systems is known as ....

(a) dynamics (b) statics (c) motion (d) acceleration

(8) In dynamics,the fps unit of work is ....

(a) $1\ g\ cm^2\ sec^{-2}$ (b) $1\ lb\ ft^2\ sec^{-3}$ (c) $1\ lb\ ft^2\ sec^{-2}$ (d) $1\ lb\ ft\ sec^{-2}$

(9) If density ρ varies from point to point in a body , then the body is said to be ....

(a) homogeneous (b) heterogeneous (c) exact (d) rigid

(10) Radial component of acceleration of a particle moving in a plane is ....

(a) $\ddot{r}+r\,\dot{\theta}^2$ (b) $\ddot{r}-r\,\dot{\theta}$ (c) $\dot{r}-r\,\dot{\theta}^2$ (d) $\ddot{r}-r\,\dot{\theta}^2$

Que.2 Answer the following. (Any Ten) 20

(1) If the plane is $V=2x^2y$, then find components of grad$V$ at the point (2,0) in the direction making an angle 45^o with Y - axis .

(P.T.O)

(2) A particle moves along the straight line and its distance from fixed point on a line is given by $x = a\cos(\mu t + z)$, where μ and z are constants. Show that the acceleration is towards the origin.

(3) A particle acted by four forces 1,3,4 and 6 lb.wt. along the sides of square taken in order, then find their resultant .

(4) State and prove the polygon law of forces .

(5) A weight of 15 lb is suspended by two ropes of 3 ft and 4 ft long fastened two points on the same horizontal line 5 ft apart, find the tension in each ropes .

(6) ABCD is a square of side 2 unit, forces 1 , 2 , 3 , 4 lb wt act along $\overline{AB}, \overline{CB}, \overline{DC}, \overline{DA}$ respectively . Find the algebraic sum of their moments about (i) A (ii) Center of a square .

(7) Prove that a plane system of forces is plane - equipollent to a single force whose components are given by $X = \sum_{i=1}^{n} X_i$, $Y = \sum_{i=1}^{n} Y_i$ and line of action is given by $xY - yX = \sum_{i=1}^{n}(x_iY_i - y_iX_i)$.

(8) Forces of magnitude 1 , 2 , 3 , 4 , $2\sqrt{2}$ are acting along the sides AB , BC , CD , DA and diagonal AC of the square $ABCD$. Show that the resultant is a couple .

(9) Find mass center of the area bounded by $y = a - x$ between the co-ordinate axes .

(10) In usual notation prove that $c^2 + s^2 = y^2$.

(11) Find the tension T for the catenary .

(12) Derive the hodograph for a particle moving in a circle with constant speed .

Que.3 (a) A scalar field is given over a plane by $V = \frac{x^2+y^2}{2x}$. What are the level curves? Show that at the point with polar co-ordinates (r, θ) , grad$V$ is inclined to the polar axis at an angle 2θ and its magnitude is $\frac{\sec^2\theta}{2}$. 5

(b) A particle moves on a straight line under a retardation kv^{m+1} , where v is the velocity at time t and k is constant . Show that 5

(i) $ks = \frac{1}{m-1}\left[\frac{1}{v^{m-1}} - \frac{1}{u^{m-1}}\right]$ (ii) $kt = \frac{1}{m}\left[\frac{1}{v^m} - \frac{1}{u^m}\right]$, where u is initial velocity .

OR

Que.3 (a) If resultant $\bar{R}$ of two forces $\bar{P}$ and $\bar{Q}$ make an angle α with first force $\bar{P}$ and β with the other force $\bar{Q}$ then prove that 4

(i) $P = \dfrac{R\sin\beta}{\sin(\alpha+\beta)}$ (ii) $Q = \dfrac{R\sin\alpha}{\sin(\alpha+\beta)}$.

(b) The maximum resultant of two forces is n-times their minimum resultant . If α is the angle between them then resultant is half of the sum of forces . Find angle α . 3

(c) Two forces $\bar{P}$ and $\bar{Q}$ acting at a point having resultant $\bar{R}$. If Q is double . then new resultant is at right angle to $\bar{P}$. Prove that $R = Q$. 3

Que.4 (a) If a system of particle is in equilibrium then prove that $\bar{F} = \bar{0}$ and $N = 0$, where $\bar{F}$ is the vector sum of the projection of all external forces on the fundamental plane and N is the sum of moments of all external forces about any line perpendicular to the fundamental plane . 3

(b) Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting at a point are in equilibrium and the angle between $\vec{P}$ and $\vec{Q}$ is doubled of angle between $\vec{P}$ and $\vec{R}$. Prove that $R^2 = Q(Q - P)$. 3

(c) A particle of weight w is suspended from a fixed point by a light string . A horizontal force H is applied to it and the particle takes up a position of equilibrium with the string inclined to a vertical . If the string breaks when the tension in it reaches at value T_0 , find the smallest value of H necessary to break the string . 4

OR

Que.4 (a) State and prove theorem of Varignon . 5

(b) A ladder of weight w rests at an angle α to the horizontal with its ends resting on a smooth floor and against a smooth vertical wall . The lower end joined by a rope to the junction of the wall and the floor , find in terms of w and α the tension of the rope,the reaction at the wall and ground. (assume that the weight of the ladder act at it's middle point) If $w = 53.4\ lb\ wt$ and $\alpha = 76.2^o$, then find tension in the rope . 5

Que.5 (a) If a rigid body is constrained to move parallel to a fixed fundamental plane , then prove that the body is in equilibrium under the action of any system of external forces plane-equipollent to zero . 3

(b) In conservative field , Prove that the force is the gradient of potential energy with sign reversed . 3

(c) Forces of magnitude 2 , 3 , 5 , 7 , $9\sqrt{2}$ are acting along the sides AB , BC , CD , DA and diagonal BD of a square $ABCD$ respectively . Taking $AB$and AD as x and y axes respectively .Find the magnitude of resultant force and equation of line of action of resultant . 4

OR

Que.5 (a) Find the potential inside and outside the spherical shell . 6

(b) Two uniform rods AB and BC each of length $2a$ are smoothly joined at a point B and rest in a virtual plane on two smooth pegs at a distance $2c$ apart. Prove that they are in equilibrium if each rod makes an angle θ with the vertical line is given by $\sin\theta = (\frac{c}{2a})^{\frac{1}{3}}$. 4

Que.6 (a) Derive differential equation of suspension bridge. Also show that it represent the equation of parabola and find its tension . 5

(b) A particle moves in a catenary $S = c\tan\psi$. The direction of its acceleration at a point makes equal angle with the tangent and normal to the path at the point. If the speed at the vertex where $\psi = 0$ is u then show that the velocity and resultant acceleration at any point are given by ue^{ψ} and $\dfrac{\sqrt{2}u^2e^{2\psi}\cos^2\psi}{c}$ respectively . 5

OR

Que.6 (a) By using intrinsic equation of the catenary prove that $y = c\left(\cosh\left(\frac{x}{c}\right) - 1\right)$. 4

(b) A uniform chain AB of length l hangs in the same horizontal line, so that the tension is n times that at lowest point . Show that the span AB must be $\dfrac{l}{\sqrt{n^2-1}}\log(n+\sqrt{n^2-1})$. 3

(c) If $\overrightarrow{V_A}$ is the velocity of the base point $A(x_A, y_A)$ and $\dot{\theta} = w$ is the angular velocity of a rigid body then find velocity $\overrightarrow{V}$ of any point of the lamina . 3

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