(A-9) Seat No: ________

No of printed pages : 3

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - V) EXAMINATION (NC)

Saturday , 14^{th} May.2016

MATHEMATICS : US05CMTH05

(Number Theory)

5U

Time : 10:30 a.m. to 01:30 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) If n is even integer then $3^n + 1$ is divisible by ....

(a) 5 (b) 2 (c) 3 (d) 4

(2) $(a\ ,\ 0) =$ .... , $\forall\ a \in \mathbb{Z}$

(a) $-a$ (b) $|a|$ (c) a (d) 0

(3) If a/bc and $(a\ ,\ b) = 1$ then ....

(a) a (b) a/c (c) b/c (d) c/a

(4) T(20) = ...

(a) 3 (b) 4 (c) 5 (d) 6

(5) P(60) = ...

(a) 120 (b) 60 (c) 60^6 (d) 60^5

(6) .... is Perfect number .

(a) 12 (b) 6 (c) 9 (d) 25

(7) If $ca \equiv ba(mod n)$ $\Rightarrow$ $a \equiv b(mod\ n)$ only if ....

(a) (b , a) = 1 (b) (b , n) = 1 (c) (a , n) = 1 (d) (c , n) = 1

(8) $ax + by = c$ has integer solution if and only if ....

(a) (a , b) = a (b) (a , b) = b (c) $(a, b)/c$ (d) $c/(a, b)$

(9) If m is prime then $\phi(m)$..................$m - 1$.

(a) $\neq$ (b) $>$ (c) $<$ (d) $=$

(10) ϕ (128) = ...

(a) 128 (b) 16 (c) 64 (d) 32

(P.T.O.)

Que.2 Answer the following (Any Ten) 20

(1) Prove that $(a-s)/(ab+st) \Rightarrow (a-s)/(at+sb)$.

(2) Find (4678 , 342) .

(3) Prove that $(a,b)=(a,b+ka)$, for $k \in Z$.

(4) If a and b are relatively prime numbers then prove that T(ab) = T(a)T(b) .

(5) Prove that $[x]+[y] \leq [x+y] \leq [x]+[y]+1$.

(6) Prove that u_m/u_{mn} .

(7) If $ca \equiv cb(mod\ n)$ and (c , n) = 1 then prove that $a \equiv b(mod\ n)$.

(8) If $a_1 \equiv b_1(mod\ n)$ and $a_2 \equiv b_2(mod\ n)$,then prove that $a_1+a_2 \equiv b_1+b_2(mod\ n)$.

(9) Is 765432 divided by 7 ?

(10) If $a_1, a_2, a_3 \ldots, a_k$ is CRS modulo m and $(a,m)=1$,then prove that $aa_1+b, aa_2+b, aa_3+b, \ \ldots \ , aa_k+b$ forms a CRS mod m,where b is any integer .

(11) State and prove Fermat's theorem .

(12) Find $\phi(243)+\phi(81)+\phi(27)+\phi(9)+\phi(3)$.

Que.3 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + \ldots.. + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 5

(b) Prove that $(a,b)=d$ iff $\exists\, x,y \in \mathbb{Z}$ such that $xa+yb=d$. 5

OR

Que.3 (a) State and prove Fundamental theorem of divisibility . 5

(b) State and prove unique factorization theorem for positive integers . 5

Que.4 (a) Prove that $S(a) < a\sqrt{a}$, $\forall\ a > 2$. 5

(b) Prove that odd prime factor of $a^{2^n}+1$ $(a>1)$ is of the form $2^{n+1}t+1$, for some integer t . 5

OR

Que.4 (a) Prove that the necessary and sufficient condition that a positive integer a can be even perfect number is $a = 2^n(2^{n+1}-1)$, $(n>1)$ and $2^{n+1}-1$ is prime . 6

(b) Prove that$u_{n+1}^2 = u_n^2 + 3u_{n-1}^2 + 2[u_{n-2}^2 + u_{n-3}^2 + \cdots + u_1]$. 4

Que.5 (a) Prove that the integer solution of $x^2+2y^2=z^2$, (x , y) = 1 can be expressed as $x = \pm(a^2-2b^2)$, $y=2ab$, $z=a^2+2b^2$. 5

(b) Prove that the positive integer solution of $x^{-1} + y^{-1} = z^{-1}$, $(x, y, z) = 1$ has and must have the form $x = a(a+b)$, $y = b(a+b)$, $z = ab$, where a , $b > 0$, $(a, b) = 1$. 5

OR

Que.5 (a) Find positive integer solution of $19x + 20y = 1909$. 5

(b) Prove that a positive integer n is divided by 3 iff the sum of its digits is divisible by 3 . 5

Que.6 (a) State and prove Chinese remainder theorem . 5

(b) Solve the system of congruences $x \equiv 2(mod\ 3)$; $x \equiv 3(mod\ 5)$; $x \equiv 2(mod\ 7)$. 5

OR

Que.6 (a) Prove that a set of k integers $a_1, a_2, a_3 \ldots, a_k$ is a complete residue system modulo m iff (i) $k = m$ (ii) $a_i \not\equiv a_j(mod\ m)$, $\forall\, i \neq j$. 5

(b) Prove that m is prime iff $\phi(m) + S(m) = mT(m)$. 5

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