(A-10) Seat No: ________ No. of Printed Pages : 2 SC

SARDAR PATEL UNIVERSITY

B.Sc.SEM-V EXAMINATION (NC)

13^{th} May 2016 , Friday

10.30 a.m. to 01.30 p.m.

US05CMTH04 (Abstract Algebra-I) Maximum Marks: 70

Q.1 Choose the correct option in the following questions, mention the correct option in the answerbook. [10]

(1) Multiplicative inverse of 5 in Z_7^* is ....
 (a) 3 (b) 6 (c) 2 (d) 1

(2) Every infinite cyclic group has exactly .... generators .
 (a) 3 (b) 1 (c) 2 (d) 4

(3) Centre of $\mathbb{Z}$ is ....
 (a) $\mathbb{Z}$ (b) 2 (c) N (d) 1

(4) In Klein 4-group $G = \{e, a, b, c\}$, $b^2 = \ldots$
 (a) e (b) b (c) c (d) a

(5) ——— is generator of group Z_n
 (a) $\bar{0}$ (b) $\bar{3}$ (c) $\bar{1}$ (d) $\bar{2}$

(6) Every cyclic group of order 4 is isomorphic to ——
 (a) Klein 4-group (b) Z (c) N (d) Z_4

(7) Order of S_4 is ....
 (a) 3 (b) 12 (c) 24 (d) 4

(8) Every group of order ——— is abelian group .
 (a) 2 (b) 5 (c) 4 (d) 6

(9) Signature of every transposition is ....
 (a) 1 (b) -1 (c) 2 (d) -2

(10) A permutation σ is said to be odd permutation if signature of σ is ....
 (a) 2 (b) -1 (c) 1 (d) -2

Q.2 Attempt the short questions. [**any ten**] [20]

(1) State first isomorphism theorem.

(2) Define Simple group and Quotient group.

(3) Prove that identity of group is unique.

(4) Find $O(2)$ in Z.

(5) State Euler's theorem.

(6) Let H be any subgroup of group G. Then prove that $aH = H \Leftrightarrow a \in H$.

(7) Prove that intersection of two subgroups of a group G is also a subgroup of G.

(8) Express the inverse of cycle (1 2 4 5 3) as a product of transpositions.

(9) Define: Group and finite group.

(PTO)

(10) Is the product of permutation of 4 symbols commutative? Justify.
(11) Define signature of the permutation.
(12) State and prove left cancelation law in group G.

Q.3(a) Let H and K be finite subgroups of group G such that HK is a subgroup of G.Then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$. [6]

(b) Prove that a non-empty subset H of a group G is subgroup iff $ab^{-1} \in H \ \forall \ a, b \in H$. [4]

OR

Q.3(c) Let G be a semigroup. Assume that,for all a,b$\in G$, the equations $ax = b$ and $ya = b$ have unique solutions in G. Then prove that G is a group. [6]

(d) Prove that intersection of any number of subgroups of a group G is also a subgroup of G. [4]

Q.4(a) Let G be a cyclic group and H is a subgroup of G. Show that H is also cyclic. [6]

(b) Let G be a group and $a, b \in G$ such that $ab = ba$. If $O(a) = n$, $O(b) = m$ with m,n relatively prime, then prove that $O(ab) = mn$. [4]

OR

Q.4(c) Prove that every subgroup of cyclic group is also cyclic. [6]

(d) If G is cyclic group of order n and $a^m = e$ for some $m \in \mathbb{Z}$ then prove that n/m . [4]

Q.5(a) Let $G = (a)$ be a finite cyclic group of order n. Then prove that the mapping $\theta : G \to G$ defined by $\theta(a) = a^m$ is an automorphism of G iff m is relatively prime to n. [6]

(b) Prove that a homomorphism $\theta : G \to G'$ of G to G' is an one-one iff $Ker\theta = \{e\}$. [4]

OR

Q.5(c) State and Prove Third isomorphism theorem. [6]

(d) Prove that a subgroup H is normal in group G iff $xH = Hx \ \forall \ x \in G$. [4]

Q.6(a) State and prove Cayley's theorem. [6]

(b) Show that $G = Z_2 \times Z_2$ is the Klein 4-group. [4]

OR

Q.6(c) Prove that S_n is non commutative group of order $n!$. [6]

(d) Prove that the set S_n of all permutation on n symbols forms a non-commutative group. [4]

—— x ——