

SARDAR PATEL UNIVERSITY
B. C. A. (I Semester) Examination
4th April 2016 (Monday)
2.30 pm – 4.30 pm
US01FBCA02 : MATHEMATICS – 1

Total Marks : 70

Q-1 Select the correct option for each of the followings: 10

- 1 If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$ then $A \cup B =$ ____
 (a) $\{1,2,3,4,6,8\}$ (b) $\{1,3\}$ (c) $\{2,4\}$ (d) None
- 2 If $A = \{a, b, c, d\}$ and $B = \{b, c, d, e\}$ then $A \oplus B =$ ____
 (a) $\{b, c, d\}$ (b) $\{a, e\}$ (c) $\{a, b, c, d, e\}$ (d) None
- 3 $A \cup U =$ _____.
 (a) A (b) \emptyset (c) U (d) None
- 4 $a * e =$ _____.
 (a) e (b) ea (c) a (d) None
- 5 $a * a^{-1} =$ _____.
 (a) $1/a$ (b) e (c) a (d) None
- 6 A semigroup with an identity element is called _____.
 (a) monoid (b) ring (c) group (d) None
- 7 If $u = (4, 7, 9)$ and $v = (2, 4, -5)$ then $u + v =$ _____.
 (a) $(6,11,4)$ (b) $(6,11, -4)$ (c) $(8,28, -45)$ (d) None
- 8 If $u = (-3, 12, -4)$ then $\|u\| =$ _____.
 (a) 12 (b) 13 (c) -13 (d) None
- 9 The observation whose frequency is highest is called _____.
 (a) mean (b) median (c) mode (d) None
- 10 _____ is the value dividing the data into two equal parts.
 (a) Mode (b) Mean (c) Median (d) None

Q-2 Do as directed : (Attempt Any 10) 20

- 1 Find the power set of $A = \{1, 2, 3, 4\}$.
- 2 prove with the help of laws : $(\emptyset \cup A) \cap (B \cup A) = A$
- 3 Calculate $4!$ = using the recursive definition.
- 4 Define left and right cancellation laws for an operation $*$ on a set S.

- 5 Let $A = \{1, 2\}$. (i) Is A closed under Addition?
(ii) Is A closed under Multiplication?
- 6 Suppose e is a left identity and f is a right identity for an operation. Show that $e=f$.
- 7 Let $u = (2, 3, -4)$ and $v = (1, -5, 8)$ then find $2u - 3v$.
- 8 Find x and y if $x(1, 1) + y(2, -1) = (1, 4)$.
- 9 Find the determinant of (i) $\begin{pmatrix} 2 & 1 \\ -4 & 6 \end{pmatrix}$ (ii) $\begin{pmatrix} a-b & a \\ a & a+b \end{pmatrix}$
- 10 The intelligence quotients (IQ's) of 10 boys is given below:
90, 120, 110, 100, 58, 53, 95, 98, 107, 110
Find the Mean IQ.
- 11 Derive the formula for Geometric Mean.
- 12 Find the median for the following data.

x_i	0	1	2	3	4	5	6	7	8
f_i	1	9	26	59	72	52	29	7	1

Q-3 Do as directed: **(10)**

- (a) Prove the following proposition: 5

$$P(n) : 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

- (b) Find a formula for the inverse of $h(x) = \frac{2x-3}{5x-7}$ 5

OR

Q-3 Do as directed: **(10)**

- (a) Consider the function $f(x) = x^2$. Find the followings: 6

(i) value of f at 5, -4 and 0.

(ii) $f(y + 2)$ and $f(x + h)$.

(iii) $[f(x + h) - f(x)] / h$.

- (b) Let a and b denote positive integers. Suppose a function 4

Q is defined recursively as follows:

$$Q(a, b) = \begin{cases} 0 & \text{if } a < b \\ Q(a - b, b) + 1 & \text{if } b \leq a \end{cases}$$

(i) Find the value of $Q(2, 3)$ and $Q(14, 3)$

(ii) What does this function do? Find $Q(5861, 7)$.

Q-4 Do as directed: (10)

- (a) Define ring. 3
- (b) Consider the set N of positive integers and let $*$ be the operation of least common multiple (l. c. m) on N . 3
- (i) Find $4*6$ and $3*5$.
- (ii) Is $(N,*)$ a semigroup? Is it commutative?
- (iii) Which elements have inverses and what are they?
- (c) Consider the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7. Find the multiplication table of G . 4
- And also find $2^{-1}, 3^{-1}, 4^{-1}, 5^{-1}$

OR

Q-4 Do as directed: (10)

- (a) Show that the left and right cancelation laws hold in G . 3
- (b) Show that $a(-b) = (-a)b = -ab$ in a ring R . 4
- (c) Consider the set Q of rational numbers and let $*$ be the operation on Q defined by $a * b = a + b - ab$. 3
- (i) Find $3*4$ and $2*(-5)$
- (ii) Is $(Q,*)$ a semigroup? Is it commutative?
- (iii) Find the identity element of $*$.

Q-5 Do as directed: (10)

- (a) If $u=(1, 2, 3, -4)$, $v=(5, -6, 7, 8)$ and $k=3$ then find: 3
- (i) $k(u \bullet v)$ (ii) $(ku) \bullet v$ (iii) $u \bullet (kv)$
- (b) Find $A^T B^T$ where $A = \begin{pmatrix} 2 & 6 & 2 \\ 5 & -1 & 3 \\ 3 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ 3
- (c) Solve by determinants : 4
- $$2x - 3y = 7$$
- $$3x + 5y = 1$$

OR

Q-5 Do as directed: (10)

- (a) Find x, y, z , and t if $A = \begin{pmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{pmatrix}$ is symmetric. 4

(b) Determine the value of k such that $\|u\| = \sqrt{39}$ where $u = (1, k, -2, 5)$ 3

(c) Find AB and BA where $A = \begin{pmatrix} 2 & 6 & 2 \\ 5 & -1 & 3 \\ 3 & 1 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ 3

Q-6 Prepare a frequency distribution of the number of letters in a word from the following paragraph (ignore punctuation marks): **(10)**

"In the beginning", said a Persian Poet, "Allah took a rose, a lily, a dove, a serpent, a little honey, a Dead Sea Apple and a handful of clay. When he looked at the amalgam – it was a woman."

Also obtain:

- (i) the number of words with letters 6 or more
- (ii) the proportion of words with 5 letters or less
- (iii) percentage of words with number of letters between 2 and 6
- (iv) the proportion of words with 6 letters or more

OR

Q-6 Find mean, median, mode, Geometric Mean and Harmonic Mean for the following data. **(10)**

x_i	130	135	140	145	146	148	149	150	157
f_i	3	4	6	6	3	5	2	1	1

ALL THE BEST