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[10]

SARDAR PATEL UNIVERSITY

BSc Sem VI Examination

Mathematics

US06CMTH04-Abstract Algebra-II

Date:03-10-22 Time:03-30 to 05-30

Q. 1 Answer the following by selecting correct choice from the options. (10)

1. _______is not an integral domain.
   a) $\mathbb{Z}$ b) $M_2(\mathbb{R})$ c) $\mathbb{Q}$ d) none
2. _______ is a ring with unity.
   a) $\mathbb{Z}$ b) $\mathbb{Q}$ c) $\mathbb{R}$ d) all of these
3. The characteristic of ring $\mathbb{Z}$ is _______
   a) 0 b) 1 c) 2 d) none
4. Quotient field of $\mathbb{Z}$ is ______
   a) $\mathbb{Z}$ b) $\mathbb{Q}$ c) $\mathbb{R}$ d) all of these
5. If $f: \mathbb{R} \to \mathbb{R}$ is a homomorphism then $f(a) =$ ____, $a \in \mathbb{R}$
   a) $\mathbb{R}$ b) 0 c) $\mathbb{Z}$ d) none
6. The ideal other than $\{0\}$ and R in a ring R is called _________
   a) proper b) improper c) trivial d) non-trivial
7. Every integral domain can be imbedded in a ______
   a) $\mathbb{Z}$ b) $\mathbb{N}$ c) Field d) Ring
8. a is divisor of b if _______, for some $c \in R$
   a) $a = bc$ b) $b = ac$ c) $c = ab$ d) $b = c$
9. If F is a field, $f(x) \in F[x], \alpha \in F$ is a root of $f(x)$ then ______
   a) $(x+\alpha)|f(x)$ b) $(x-\alpha)|f(x)$ c) $f(x)|(x-\alpha)$ d) $f(x)|(x+\alpha)$
10. If $R = \mathbb{Z} + i\mathbb{Z}$, $f(x) = 2x^2 - (1+i)x - 2$ then content of f is _______
    a) $2 - i$ b) $2 + i$ c) $1 - i$ d) $1 + i$

Q.2 Do as directed. (8)

1) True or False: $\mathbb{Z}_5$ is an integral domain.
2) Fill in the blank: Every subring is an ______ in a ring(ideal/not an ideal).
3) True or False: $\mathbb{Z}$ is a not a field.
4) True or False: Field has no proper ideals.
5) Fill in the blank: If I is an ideal in ring R and $a + I = I$ then ________$(a = I/a \in I)$.
6) Fill in the blank: The polynomial $1 + x + x^2 + x^3 + x^4 \in \mathbb{Q}$ is _____(irreducible/ reducible.

(P.T.O.)

7) True or False: If F is a field, degree of $f(x) \in F[x]$ is n then $f(x)$ has at least n distinct roots.

8) True or False: If $f(x) = x^4 - 2x^2 + 1$ and $g(x) = x^3 + x + 1$ are polynomials in $\mathbb{R}[x]$ then $deg(fg)$ is 12.

Q. 3 Answer any TEN. **(20)**

1) Define Ring with unity with example.
2) Define Characteristic of ring.
3) Find Characteristic of $\mathbb{Z}_3$.
4) In a ring $(R, +, \cdot)$ prove that $a \cdot 0 = 0 \cdot a = 0$.
5) Define Simple Ring.
6) Find invertible elements of ring $\mathbb{Z}$, if any.
7) What are improper ideals?
8) Define Associates in a ring.
9) Prove that $1 + 2i$ and $2 - i$ are associates in $\mathbb{Z} + i\mathbb{Z}$.
10) Find all roots of $x^3 + 5x$ in $\mathbb{Z}_6$.
11) Define content of polynomial.
12) Prove that $1 + 3i$ divides 10 in $\mathbb{Z} + i\mathbb{Z}$.

Q.4 Attempt any FOUR. **(32)**

1) Prove that every finite integral domain is a field.
2) Prove that the only isomorphism of $\mathbb{R}$ onto $\mathbb{R}$ is the identity map $I_{\mathbb{R}}$.
3) Prove that $m\mathbb{Z}$ is an ideal in $\mathbb{Z}$ for $m \geq 0$.
4) Prove that every Simple Ring need not be a field.
5) Let $R = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$, show that $1 + 2\sqrt{-5}$ and 3 are relatively prime.
6) Show that the ring of Gaussian integers is an Euclidean domain.
7) Let R be a commutative ring and $f(x), g(x) \in R[x]$ then prove that
8) State and prove Eisenstein's criterion.

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