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[A-14]

Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-VI : Examinations : 2022-23 [NC]

Subject : Mathematics US06CMTH03 Max. Marks : 70

Topology

Date: 01/10/2022, Saturday Timing: 03.30 pm - 05.30 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. **10**

[1] Any topology on a non-empty set is ______ the indiscrete topology on that set.
[A] coarser than [B] finer than [C] non-comparable [D] none

[2] In a topological space $(X, \mathcal{T})$, every $\mathcal{T}$-open set
[A] can not be a neighbourhood of all its points
[B] is a neighbourhood of all its points
[C] is $\mathcal{T}$-closed also
[D] none

[3] In a topological space $(X, \mathcal{T})$, a neighbourhood of a point is
[A] $\mathcal{T}$-open [B] $\mathcal{T}$-closed [C] either open or closed [D] none

[4] If A is a closed set in a topological space then
[A] $A \subset A'$ [B] $A^- \neq A$ [C] $A = A'$ [D] $A' \subset A$

[5] Minimum number of open as well as closed subsets in a topological space is
[A] 0 [B] 1 [C] 2 [D] 3

[6] If G is an open set then every point in G is
[A] an interior point of G
[B] a cluster point of G
[C] an interior point as well as cluster point of G
[D] neither an interior point nor a cluster point of G

[7] Every non-empty and bounded above subset of R posseses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

[8] Every non-empty and bounded below subset of R posseses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

[9] In the relativized topology of $\mathcal{U}$-topology of $\mathbb{R}$ the subset $[1, 10]$ is
[A] Compact [B] Disconneceted [C] Unbounded [D] none

[10] A Hausdorff Space is also called a
[A] T_1 [B] T_2 [C] T_3 [D] Regular Space

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false **08**

[1] If (X, T) is a topological space then every subset of X is a closed set(True/False?)

[2] If (X, T) is a topological space then every subset of X must be either a closed set or an open set(True/False?)

[3] Every set has atleast one interior point (True/False?)

[4] If closure of a set is same as the set then the set must be closed (True/False?)

[5] In a $(R, \mathcal{U})$ the subspaces $(0, 1)$ ans $(0, 100)$ are homeomorphic (True/False).

[6] Let f be a function from a topological space (X, T) into another topological space (Y, V) such that $A \subset X$ is connected but $f(A)$ is not connected in Y. Then f is not continuous on X. (True/False).

[7] T_1 Space and Regular spaces are same.(True/False?)

[8] A T_1 space is a Housdorff space. (true/False?)

Q: 3. Answer ANY TEN of the following. **20**

[1] Define : (i) Usual Topology of $\mathbb{R}$ (ii) Open Set

[2] Give an example of a Door Space

[3] If $X = \{a, b, c\}$ then find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ for X such that $\mathcal{T}_1 \subsetneq \mathcal{T}_2 \subsetneq \mathcal{T}_3$

[4] Find $\mathcal{U}$-closures of the sets $\mathbb{R}$ and $\emptyset$.

[5] For any topologies $\mathcal{T}$ and Ψ of $\mathbb{R}$ show that the mapping $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2, \forall x \in \mathbb{R}$, is $\mathcal{T}$-Ψ continuous

[6] Find $\mathcal{U}$-closures of the sets $\mathbb{R}$ and $\emptyset$.

[7] Let $f : [0, 1] \to R$ be continuous on $[0, 1]$. Is $f([0, 1])$ connected?

[8] State the Least Upper Bound property of R

[9] Prove that a continuous image of connected space is connected

[10] Give an example of a T_1-space that is not a T_2-space

[11] Prove that the space $(R, \mathcal{U})$ is a T_2-space.

[12] Prove that every metric space is a Hausdorff space

Q: 4. Attempt any FOUR of the following. **32**

[1] If $(X, \mathcal{T})$ is a topological space and $\{F_\alpha / \alpha \in \Lambda\}$ is any collection of $\mathcal{T}$-closed subsets of X then prove that $\bigcap\{F_\alpha / \alpha \in \Lambda\}$ is a $\mathcal{T}$-closed set

[2] Let J be the set of all integers and $\mathcal{J}$ be a collection of subsets G of J where $G \in \mathcal{J}$ whenever $G = \emptyset$ or $G \neq \emptyset$ and $p, p \pm 2, p \pm 4, ..., p \pm 2n, ...$ belong to G whenever $p \in G$.Prove that $\mathcal{J}$ is a topology for J

[3] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X and A' be the set of all cluster points of A. Prove that A is $\mathcal{T}$-closed iff $A' \subset A$

[4] Let $(X, \mathcal{T})$ be a topological space and A be a subset of X. Prove that $A \cup A'$ is $\mathcal{T}$-closed

[5] Show that a relative topology satisfies all the conditions for becoming a topological space

[6] Assuming that connectedness is a topological property prove that $(R, \mathcal{U})$ and $(R, \mathcal{G})$ are not homeomorphic where $\mathcal{U}$ is usual topology for R and $\mathcal{G}$ is defined as follows $G \in \mathcal{G}$ if either G empty or it is a nonempty subset of R such that for every $p \in G$ there is some $H = \{x \in R / a \leqslant x < b\}$ for $a < b$ such that $p \in H \subset G$.

[7] Prove that the space $(R, \mathcal{U})$ is a T_3-space.

[8] If $(X, \mathcal{T})$ is a compact space, and if f is a $\mathcal{T} - \psi$ continuous mapping of X into R, then prove that f is bounded.

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