SEAT No.__________

[17]

No of printed page : 2

SARDAR PATEL UNIVERSITY

B.Sc.(SEMESTER - VI) EXAMINATION - 2022

Friday, 30^{nd} September 2022 MATHEMATICS: US06CMTH02

(Complex Analysis)

Time : 03:30 p.m. to 05:30 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) Domain of $f(z) = \dfrac{z}{z+\bar{z}}$ is ....

(a) $\{z \in \mathbb{C} \ / \ Imz = 0\}$ (b) $\{z \in \mathbb{C} \ / \ Rez \neq 0\}$ (c) $\{z \in \mathbb{C} \ / \ Imz \neq 0\}$

(d) $\{z \in \mathbb{C} \ / \ Rez \neq 1\}$

(2) $f(z) = (x^2 - y^2 - 2y) + i(2x - 2xy)$ can be expressed as $f(z) =$........................

(a) $\bar{z}^2 + 2z$ (b) $\bar{z}^2 + iz$ (c) $\bar{z}^2 - 2iz$ (d) $\bar{z}^2 + 2iz$

(3) $\lim\limits_{z\to\infty} f(z) = \infty$ iff .... $= 0$

(a) $\lim\limits_{z\to\infty} \dfrac{1}{f(z)}$ (b) $\lim\limits_{z\to 0} \dfrac{1}{f\left(\frac{1}{z}\right)}$ (c) $\lim\limits_{z\to 0} f\left(\dfrac{1}{z}\right)$ (d) $\lim\limits_{z\to 0} \dfrac{1}{z}$

(4) If $f(z) = 2x + iy^2x$ then f is differentiable at ....

(a) 2 (b) 1 (c) 0 (d) none of these

(5) $f(z) = \dfrac{z^3 + i}{(z^2 - 3z + 2)}$ is analytic in ....

(a) $\{\pm\sqrt{1} \ , \ \pm 2\}$ (b) $\mathbb{C} - \{1 \ , \ 2\}$ (c) $\mathbb{C} - \{3 \ , \ \pm 2\}$ (d) $\{ 1 \ , \ 2 \}$

(6) If C-R equations are not satisfied at z_0 then f(z) is .... at z_0 .

(a) differentiable (b) not differentiable (c) need not be differentiable (d) none of these

(7) if e^z is purely imaginary then $Imz =$ .... , $n \in \mathbb{Z}$.

(a) $(2n+1)\pi$ (b) $2n\pi$ (c) π (d) $(2n+1)\pi/2$

(8) $\cos iy =$ ....

(a) $\cosh y$ (b) $i\cos y$ (c) $-\cosh y$ (d) $i\cosh y$

(9) Image of $x > 0$ under the transformation $w = (1+i)z$ is ....

(a) $u < v$ (b) $v < u$ (c) $u < -v$ (d) $u > -v$

(10) Fixed point of $w = \dfrac{z-1}{z+1}$ are ....

(a) $\pm\ i$ (b) i (c) -1 (d) 3

Que.2 Write TRUE or FALSE. [8]

(1) Cartesian form of $f(z) = z^2$ is $f(z) = x^2 - y^2 + i2xy$

(2) $\lim\limits_{z\to z_0} f(z) = \infty$ iff $\lim\limits_{z\to z_0} \dfrac{1}{f(z)} = 0$

(3) Singular point of $f(z) = \dfrac{z^3+4}{(z^2-3)}$ is $z = \pm\sqrt{3}i$

(P.T.O.)

(4) If $u(x,y) = y^3 - 3x^2y$ then $u_{xx} + u_{yy} = 1$

(5) $exp(z + \pi i) = e^z$.

(6) e^z is periodic function with period $2n\pi i$, $n \in \mathbb{Z}$

(7) The image of line $x = c_1$, $c_1 \neq 0$ under the transformation $w = 1/z$ is circle .

(8) If $T(z) = \dfrac{az+b}{cz+d}$, $(ad - bc \neq 0)$.Then $\lim_{z\to\infty} T(z) = c/a$, $if\ c \neq 0$.

Que.3 Answer the following (Any ten) 20

(1) Express $f(z) = z^3 + z + 1$ in cartesian form.

(2) Evaluate $\lim_{z\to 0} \dfrac{z}{\bar{z}}$, if possible.

(3) By using definition, prove that $\dfrac{d}{dz}(z^n) = nz^{n-1}$ for all $n \in \mathbb{Z}$.

(4) Check that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is entire or not.

(5) Prove that $e^{-y}\sin x$ and $-e^{-y}\cos x$ are harmonic at each point of domain of xy-plane.

(6) Prove that $f(z) = e^{i\bar{z}}$ is nowhere analytic.

(7) Prove that $\overline{exp(iz)} = exp(i\bar{z})$ iff $z = n\pi$, $n \in \mathbb{Z}$.

(8) Prove that $|\cosh z|^2 = \sinh^2 x + \cos^2 y = \cosh^2 x - \sin^2 y$.

(9) Evaluate $log(-1 - \sqrt{3}i)$ and $log(ei)$.

(10) Find the image of $0 < x < 1$, $0 < y < 1$ under the transformation $w = iz$.Also sketch the region.

(11) Find the image of $0 < y < \dfrac{1}{2c}$ under the transformation $w = 1/z$.Also sketch the region.

(12) Find the image of $a \le x \le b$; $0 \le y \le \pi$ under the transformation $w = e^z$.Also sketch the region.

Que.4 Attempt the following (Any FOUR) [32]

(1) If $f(z) = \dfrac{x^3y(y - ix)}{z(x^6 + y^2)}$, $z \neq 0$, $f(0) = 0$
(i) Is $f(z)$ continuous at 0 ? (ii) Is $f(z)$ differentiable at 0 ?

(2) If f and g are differentiable then prove that fg and f/g are differentiable .

(3) Let $f(z) = u(x,y) + iv(x,y)$. If $f'(z)$ exist at $z_0 = x_0 + iy_0$ then prove that the first order partial derivatives of u and v must exist at (x_0, y_0) and they satisfies the Cauchy-Reimann equations $u_x = v_y$; $u_y = -v_x$ *at* (x_0, y_0). Under which condition converse of above statement hold.Prove it.

(4) Find a harmonic conjugate $v(x,y)$ for $u(x,y) = 2x - x^3 + 3xy^2$.

(5) Prove that $\tanh^{-1} z$ is multiple valued function .

(6) Solve the equation $\sinh z = i$ and $e^z = \sqrt{3} - i$

(7) Prove that all linear fractional transformation that maps the upper half plane $Imz > 0$ on to the open disk $|w| < 1$ and the boundary $Im\ z = 0$ on to the boundary of $|w| = 1$ is given by $w = e^{i\alpha}\left[\dfrac{z - z_0}{z - \overline{z_0}}\right]$, $(Im\ z_0 > 0)$. Also prove the converse .

(8) Find linear fractional transformation that maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ on to $w_1 = 1$, $w_2 = i$, $w_3 = -1$ respectively .

—x—