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[24]

SARDAR PATEL UNIVERSITY(B.Sc. Sem.6 Examination)
MATHEMATICS - US06CMTH24
Riemann Integration & Series of Functions
27th June 2022, Monday

Time: 10:00 TO 12:00 p.m. **Maximum Marks: 70**

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. **[10]**

1 If $P^* = P_1 \cup P_2$ then $P^*$is ________than P_1.
(a)finer (b) union (c) intersection (d) refinement

2 If $P^*$is a refinement of P,then for a bounded function f, $L(P^*, f)$ ________ $L(P, f)$
(a) $<$ (b) $\leq$ (c) $>$ (d) $\geq$

3 $\text{Inf}(U(P,f)) =$ ______
(a) $\int_a^{\bar{b}} f\,dx$ (b) $\int_{\underline{a}}^{b} f\,dx$ (c) $\int_a^b f\,dx$ (d) none of these

4 $\sum_{i=1}^{n} f(t_i)\Delta x_i =$ ________
(a)$U(P,f)$ (b) $L(P,f)$ (c) $S(P,f)$ (d) none of these

5 A bounded function f, having a ________number of points of discontinuity on $[a,b]$ is integrable on $[a,b]$
(a) infinite (b) finite (c) no (d) only one

6 $f \in \mathbb{R} \Rightarrow \lim_{\mu(P)\to 0} S(P,f)$ ___________
(a) $< \varepsilon$ (b) $> \varepsilon$ (c) does not exist (d) exists

7 If f and g be two positive functions in $[a,b]$ such that $\lim_{x\to a^+} \frac{f(x)}{g(x)} = l$ then $\int_a^b f\,dx$ and $\int_a^b g\,dx$ converge and diverge together at a if $l =$_________.
(a)1 (b)0 (c) $+\infty$ (d)$-\infty$

8 The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges iff _________
(a) $n \leq 1$ (b) $n \leq 0$ (c) $n < 1$ (d) $n < 0$

9 The series $\sum \frac{1}{2n^{\frac{1}{2}(p+q)}}$ converges for $p+q$________2.
(a) $\geq$ (b) $>$ (c)$\leq$ (d) $<$

10 The series $\sum \frac{\alpha}{n^p}$ $(\alpha \geq 0)$ converges for p ________1.
(a) $\geq$ (b) $>$ (c)$\leq$ (d) $<$

Q.2 Answer the following.(True/False) **[08]**

1 $\int_a^b f\,dx$ exists, implies that function f is unbounded and integrable over $[a,b]$

2 If $P^* \supset P$ then $P^*$is refinement of P

3 For a function f, $\lim_{\mu(P)\to 0} S(P,f) = \int_a^b f\,dx$

4 If a function f is monotonic on $[a,b]$,then it is integrable on $[a,b]$

5 Every absolutely convergent integral is convergent.

6 $\int_0^\infty \frac{sin^2x}{x^2} dx$is convergent.

7 The series $\sum(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every interval

8 The series $\sum(-1)^n \frac{x^2+n}{n^2}$ converges absolutely for any value of x in $[a, b]$

Q.3 Answer ANY TEN of the following. **[20]**

1 In usual notations prove that, $m(b-a) \leq L(P, f) \leq U(P, f) \leq M(b-a)$

2 State second form of Darboux's theorem.

3 Define: Upper sum

4 State first fundamental theorem of integral calculus

5 Evaluate $\int_0^2 [x] \ dx$

6 State Generalised First Mean Value theorem.

7 Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x}}$

8 Test the convergence of $\int_0^{\frac{\pi}{2}} \frac{\sin x}{x^p}$

9 Examine the convergence of $\int_2^\infty \frac{2x^2dx}{x^4-1}$

10 Show that $\sum \frac{a_n}{n^x}$ converges uniformly in $[0,1]$ if $\sum a_n$ converges.

11 Prove that the series $\sum \frac{(-1)^n}{n} |x|^n$ is uniformly convergent in $-1 \leq x \leq 1$

12 Define: Uniform convergence of a sequence of functions on an interval.

Q-4 Answer ANY FOUR of the following. **(32)**

1 Show that$(3x + 1)$ is integrable over $[0,1]$ and $\int_0^1 (3x + 1)dx = \frac{5}{2}$

2 State and prove Darboux's theorem

3 Show that a function f is integrable over $[a, b]$ iff for $\varepsilon > 0 \ \exists \ \delta > 0 \ni$ if P, P' are any two partitions of $[a, b]$ with mesh less than δ, then $|S(P, f) - S(P', f)| < \varepsilon$

4 Prove that every continuous function is integrable.

5 Show that $\int_0^1 x^{m-1}(1-x)^{n-1} \ dx$ exists if and only if m, n are both positive.

6 Show that the integral $\int_0^{\frac{\pi}{2}} \log \sin x \ dx$ is convergent and hence evaluate it.

7 Prove that the limit of integrals is not equal to the integral of the limit.

8 State and prove Cauchy's criterion for uniform convergence of a sequence of functions.

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