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Seat No.________ No. of Printed Pages: 2

SARDAR PATEL UNIVERSITY(B.Sc. Sem.6 Examination)
MATHEMATICS - US06CMTH23 – Linear Algebra
Saturday, 25th June 2022

Time: 10:00 TO 12:00 **Maximum Marks: 70**

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. [10]

1 If V has a basis of n elements then every set of p-vectors with p_______n is L.D.
(a) $<$ (b) $\leq$ (c) $>$ (d) $\geq$

2 $dim.\mathcal{P}_2 =$ _________.
(a) 0 (b)1 (c) 2 (d) 3

3 $dim.(xy - plane + yz - plane) =$______.
(a) 4 (b) 3 (c) 2 (d) 1

4 If U is finite-dimensional, then $dim.R(T)$_______$dim.U$
(a) $<$ (b) $\leq$ (c) $>$ (d) $\geq$

5 Let $T: U \to V$ be a linear map. Then, if $[u_1, u_2, \dots, u_n] = U$ then $[T(u_1), T(u_2), \dots, T(u_n)]$ = __________
(a) $N(T)$ (b)$K(U)$ (c)$R(T)$ (d)$T(U)$

6 A linear map $T: U \to V$ is one-one and onto, then it is called __________.
(a) isomorphism (b) homomorphism (c) singular (d)none of these

7 The range of an $m \times n$ matrix is the ________of its column vectors.
(a) total (b) span (c) multiplication (d)none of these

8 A square matrix that is not invertible is called a _____matrix
(a) singular (b) non- singular (c) zero (d) identity

9 If u and v are vectors in an Inner product space V, $(\alpha u) \cdot v =$_______
(a) αuv (b) $\alpha \cdot (uv)$ (c) $\alpha(uv)$ (d) $\alpha(u \cdot v)$

10 If u be a vector in an Inner product space V, $\bar{0} \cdot u =$_______.
(a) $u \cdot 0$ (b) $u\bar{0}$ (c) 0 (d) $\bar{0}$

Q.2 Answer the following.(True/False) [08]

1 The set of all functions $f \in \mathcal{C}[0,1]$ such that f has a local maxima at $x = \frac{1}{2}$ is a vector space.

2 The span of x-axis and the plane $x + y = 0$ in V_3 is xy-plane.

3 There exists a linear transformation $T: V_2 \to V_4$ such that $T(0,0) = (1,0,0,0)$

4 Let $T: V_3 \to V_3$ be a linear and onto map then T is one-one.

5 Matrix associated with linear map $T: V_2 \to V_4$ is of order (2×4)

6 To every linear transformation there corresponds a unique matrix.

7 A set of vectors is said to be orthogonal if a pair of distinct vectors of the set is orthogonal.

8 If u and $v$belong to an Inner product space V and $v \neq \bar{0}$then the vector $\frac{u \cdot v}{||v||^2}$ v is called the vector projection of u and along v.

Q.3 Answer ANY TEN of the following. **[20]**

1 For any vector space V , Prove that $(-1)u = -u, \ \forall\, u \in V$

2 Check whether the subset $\{(x_1, x_2, x_3)/x_1 + x_2 + x_3 \geq 0\}$ of V_3 is a subspace ?

3 Let V be any vector space. Then the set $\{v\}$ is L.D. iff $v = \bar{0}$

4 If U and V are finite dimensional vector spaces of the same dimension, then prove that a linear map $T: U \to V$ is one-one iff it is onto.

5 Check linearity of a map $T: V_1 \to V_3$ defined by $T(x) = (x, x^2, x^3)$

6 Let $T: U \to V$ be linear map, then prove that$N(T)$ is a subspace of U.

7 Let a Linear map $T: V_2 \to V_3$ be defined by $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$. Find matrix associated with T relative to basis $B_1 = \{e_1, e_2\}$ and $B_2 = \{f_1, f_2, f_3\}$

8 If U and V are finite dimensional vector spaces then prove that$dim.\big(L(U, V)\big) = dim.\, U \times dim.\, V$

9 Prove that the columns of a square matrix A are L.I. iff its row are L.I

10 In an inner product space V, prove that $||\alpha u|| = |\alpha|\ ||u||$

11 Prove that any Orthogonal set of non-zero vectors in an Inner product space is L.I.

12 Define: Orthonormal set of vectors.

Q-4 Answer ANY FOUR of the following. (32)

1 Let R^+ be the set of all positive real numbers. Define the operations of addition and scalar multiplication as follow:

$u + v = u \cdot v$ for all $u, v \in R^+$

$\alpha u = u^\alpha$ for all $u \in R^+$ and real scalar α

Prove that R^+ is a real vector space.

2 Let V be any vector space. Then the set $\{v_1, v_2, v_3\}$ is L.D. iff v_1, v_2 and $v_3$are coplanar

3 State and prove Rank-Nullity theorem.

4 Prove that a linear map $T: V_3 \to V_3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_1 + x_2, x_3)$ is non-singular. Also find T^{-1}

5 Determine a linear map $T: V_3 \to V_2$ corresponding to matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ relative to $B_1 = \{(1,1,1), (1,2,3), (1,0,0)\}$, $B_2 = \{(1,0), (2, -1)\}$

6 Find range, rank, Kernel, nullity of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$

7 In an Inner product space V, prove that

(i) $|u \cdot v| \leq ||u||\ ||v||$

(ii) $||u + v|| \leq ||u|| + ||v||$

8 Orthonormalise the set of L.I. vectors $\{(1,0,1,1), (-1,0,-1,1), (0,-1,1,1)\}$ of V_4

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