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SARDAR PATEL UNIVERSITY
BSc Sem VI Examination - 2022
Mathematics
US06CMTH22-Ring Theory

Date: 24-6-2022, Friday Time : 10 am To 12 pm

Q. 1 Answer the following by selecting correct choice from the options. (10)

1) _______ is not an integral domain.

a. $\mathbb{Z}$ b. $\mathbb{Z}_6$ c. $\mathbb{Z}_7$ d. $\mathbb{Q}$

2) ___and ___ are regular elements of $\mathbb{Z}$.

a. 1 and -1 b. 0 and 1 c. -1 and 0 d. none

3) $\mathbb{Z}_p$ is a field if p is a ______.

a. prime b. composite c. any number d. none

4) An ideal $\{0\}$ is _________ ideal in ring R.

a. proper b. improper c. proper and improper d. none

5) If I is an ideal in ring R with unity then unit element in R/I is ____

a. 0 b. R c. 1 d. $1+I$

6) ________ is an ideal in $\mathbb{Z}_6$.

a. $\{\bar{0},\bar{3}\}$ b. $\{\bar{0},\bar{2}\}$ c. $\{\bar{0},\bar{4}\}$ d. none

7) Every ________ has unit element.

a. Integral Domain b. Ring c. Euclidean Domain d. Commutative Ring

8) Let R be an Euclidean domain, $a,b \in R$, a is proper divisor of b then $d(b)$___$d(a)$.

a. $=$ b. $\leq$ c. $<$ d. $>$

9) If F is a field, $f(x) \in F[x], \alpha \in F$ is a root of $f(x)$ then ______

a. $(x-\alpha)|f(x)$ b. $(x+\alpha)|f(x)$ c. $f(x)|(x-\alpha)$ d. $f(x)|(x+\alpha)$

10) If $R = \mathbb{Z} + i\mathbb{Z}$, $f(x) = 5x^2 + 5x - (2+i)$ then content of f is ________.

a. $1+i$ b. $1-i$ c. $2+i$ d. $2-i$

Q.2 Do as directed. **(8)**

1) Fill in the blank: ________ is a field. ($\mathbb{Z}$/ $\mathbb{Q}$)
2) Fill in the blank: The characteristic of ring $\mathbb{Z}$ is ________.(0/1)
3) True or False: Every subring is an ideal in a ring.
4) True or False: Field has no proper ideals.
5) Fill in the blank: Every irreducible element in unique factorization domain is ________.(prime/not prime)
6) True or False: If $f(x) = 3x^5 + 2x^3 + 1$ and $g(x) = x^2 + 1$ are polynomials in $\mathbb{Z}[x]$ then $deg(fg)$ is 7.
7) True or False: The polynomial $f(x) = 3x^3 - 2x^2 + 6x + 9$ is primitive polynomial.
8) Fill in the blank: The polynomial $x^2 - 3 \in \mathbb{Q}[x]$ is ________(reducible/irreducible).

Q.3 Answer any TEN. **(20)**

1) Define a ring without zero divisor.
2) Find Characteristic of $\mathbb{Z}_5$.
3) Let R be a ring then prove that, $for\ all\ a, b \in R, a(-b) = (-a)b = -(ab)$.
4) Find all regular elements of $\mathbb{Z}_{20}$.
5) Prove that $3\mathbb{Z}$ is an ideal in ring $\mathbb{Z}$.
6) Define simple ring with illustration.
7) Find $\mathbb{Z}/_{5\mathbb{Z}}$.
8) What is the ideal generated by 1 in the ring $(\mathbb{Z}, +, \cdot)$?
9) Define an Euclidean Domain.
10) Prove that $1 + 3i$ divides 10 in $\mathbb{Z} + i\mathbb{Z}$.
11) Find all roots of $x^3 + 5x$ in $\mathbb{Z}_6$.
12) If F is a field then prove that $F[x]$ is an U.F.D..

Q.4 Attempt any FOUR. **(32)**

1) Find the regular elements in the ring $R = \{a + b\sqrt{-5}\ |a, b \in \mathbb{Z}\}$.
2) Prove that every finite integral domain is a field.
3) Prove that every commutative simple ring with unit element is a field.
4) Prove that P is prime ideal of ring $\mathbb{Z}$ iff $P = 0$ or $P = p\mathbb{Z}$ for some prime number p.
5) Prove that Every UFD need not be a PID.
6) Prove that any two elements of U.F.D. have a GCD.
7) State and prove Eisenstein's criterion.
8) If p is prime then prove that $f(x) = 1 + x + x^2 + x^3 + \cdots \ldots \ldots + x^{p-1} \in \mathbb{Q}[x]$ is irreducible.