SEAT No.__________

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SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - VI ) EXAMINATION - 2022
Thursday, 23rd June 2022 MATHEMATICS: US06CMTH21
( Complex Analysis )

Time : 10:00 a.m. to 12:00 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. [10]

(1) Domain of $f(z)=\dfrac{z}{z-\bar{z}}$ is $\{z\in\mathbb{C}\ /.....................\}$

(a) $Imz\neq 0$ (b) $Rez\neq 0$ (c) $Imz=0$ (d) $Rez\neq 1$

(2) $\lim\limits_{z\to\infty} f(z)=w_0$ iff ................................ $=w_0$

(a) $\lim\limits_{z\to\infty}\dfrac{1}{f(z)}$ (b) $\lim\limits_{z\to 0}\dfrac{1}{f(z)}$ (c) $\lim\limits_{z\to 0} f\left(\dfrac{1}{z}\right)$ (d) $\lim\limits_{z\to 0}\dfrac{1}{z}$

(3) Cartesian form of $f(z)=\bar{z}^2+2iz$ is $f(z)=$ .....................

(a) $x^2-y^2-2y+i2x(1-y)$ (b) $(x^2+y^2-2y)+i2x(1-y)$

(c) $(x^2+y^2+2y)+i2x(1-y)$ (d) $x^2-y^2-2y-i2x(1-y)$

(4) Singular point of $f(z)=\dfrac{z^3+4}{(z^2+3)(z^2+1)}$ are $z=$ ....................

(a) $\sqrt{3}$ , $i$ (b) $\pm\sqrt{3}$ (c) $\pm\sqrt{3}$ , $\pm i$ (d) $\pm\sqrt{3}\,i$ , $\pm i$

(5) If $u(x,y)=y^3-3x^2y$ then .....................

(a) $u_{xx}+u_{yy}=0$ (b) $u_{xx}+u_{xy}=0$ (c) $u_{xx}+u_{yy}=1$ (d) none of these

(6) $\overline{exp(iz)}=exp(i\bar{z})$ iff $z=$ ................ , $n\in\mathbb{Z}$.

(a) $2n\pi i$ (b) $2n\pi$ (c) $n\pi$ (d) $(2n+1)\pi$

(7) if $e^z$ is purely imaginary then $Imz=$ ............... , $n\in\mathbb{Z}$ .

(a) $(2n+1)\pi$ (b) $2n\pi$ (c) $\pi$ (d) $(2n+1)\pi/2$

(8) $i\sin iy=$ ..............

(a) $-\sinh y$ (b) $isinhy$ (c) $-isinhy$ (d) $cosiy$

(9) Image of $x>0$ under the transformation $w=(1+i)z$ is ........................

(a) $u<v$ (b) $v<u$ (c) $u<-v$ (d) $u>-v$

(10) Fixed point of $w=\dfrac{z-1}{z+1}$ are .....................

(a) $\pm\ i$ (b) $i$ (c) $-1$ (d) $3$

Que.2 Write TRUE or FALSE. [8]

(1) $f(z)=|z|^2$ is not differentiable at z = 1.

(2) $\lim\limits_{z\to\infty} f(z)=\infty$ iff $\lim\limits_{z\to z_0}\dfrac{1}{f(z)}=0$

(3) Singular point of $f(z)=\dfrac{2z}{z(z^2+1)}$ are $z=0$ , $\pm i$



(P.T.O.)

(4) If C-R equations are not satisfied at $z_0$ then $f(z)$ is differentiable at $z_0$ .

(5) $exp(z + \pi i) = e^z$.

(6) $e^z$ is periodic function with period $2n\pi i$ , $n \in \mathbb{Z}$

(7) The image of line $x = c_1$ , $c_1 \neq 0$ under the transformation $w = 1/z$ is circle .

(8) If $T(z) = \dfrac{az+b}{cz+d}$ , $(ad - bc \neq 0)$.Then $\lim_{z\to\infty} T(z) = c/a$ , $if\ c \neq 0$.

Que.3 Attempt the following ( Any TEN ) [20]

(1) Express $f(z) = (x^3 - 3xy^2 + x + 1) + i(3x^2y + y - y^3)$ in terms of $z$ and simplify the result, where $z = x + iy$ .

(2) Evaluate $\lim_{z\to 0} \dfrac{\bar{z}^2}{z}$.

(3) Prove that every polynomial is differentiable at each point.

(4) Define Analytic function and Singular point.

(5) Check that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is entire or not .

(6) Prove that $f(z) = xy + iy$ is nowhere analytic.

(7) Prove that $e^{-y} \sin x$ and $-e^{-y} \cos x$ are harmonic at each point of domain of xy-plane.

(8) Prove that $\dfrac{e^{z_1}}{e^{z^2}} = e^{z_1 - z_2} \quad \forall \quad z_1, z_2 \in \mathbb{C}$.

(9) Prove that $Log(-ei) = 1 - \dfrac{\pi}{2} i$.

(10) Find $\dfrac{d}{dz}(\operatorname{sech} z)$.

(11) Prove that $w = z + B$ ,where B is complex constant, gives a translation by means of vectors representing B.

(12) Find image of $x > 1$ , $0 < y$ under the transformation $w = 1/z$ .Also sketch the region.

Que.4 Attempt the following ( Any FOUR ) [32]

(1) Prove that $f(z) = |z|^2$ is differentiable only at z=0.Also prove that $f'(0) = 0$.

(2) If f and g are differentiable then prove that f/g is differentiable and $\left(\dfrac{f}{g}\right)'(z) = \dfrac{g(z)f'(z) - f(z)g'(z)}{g(z)^2}$ ,if $g(z) \neq 0$.

(3) Let $f(z) = u(x, y) + iv(x, y)$ and $f'(z)$ exist at $z_0 = x_0 + iy_0$.Prove that the first order partial derivatives of $u$ and $v$ must exist at $(x_0, y_0)$ and they satisfies the Cauchy-Reimann equations $u_x = v_y$ ; $u_y = -v_x\ at\ (x_0, y_0)$.Also prove that $f'(z) = u_x + iv_x$ where $u_x\ and\ v_x$ are evaluated at $(x_0, y_0)$.

(4) Prove that $u(x, y) = x^2 - y^2$ is harmonic in some domain and find a harmonic conjugate v(x,y) for u(x,y).Also find corresponding analytic function f(z).

(5) Prove that $sin^{-1}z = -ilog[iz + \sqrt{1 - z^2}]$. Also using it find $sin^{-1}(-i)$

(6) Find all roots of $coshz = 1/2$. Also Prove that $Log(-1 + i) = \dfrac{1}{2}ln2 + 3\dfrac{\pi}{4}i$.

(7) Prove that composition of linear fractional transformation is also linear fractional transformation

(8) Discuss the image of $w = (i + 1)z + 2$ . Hence sketch the rectangle $1 \leq x \leq 2$ , $1 \leq y \leq 4$ and its image.

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