SEAT No.________

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Sardar Patel University, Vallabh Vidyanagar

B.Sc. - Semester-VI : Examinations : 2022-23 [NC]

Subject : Mathematics US06CMTH03 Max. Marks : 70

Topology

Date: 25/06/2022, Saturday Timing: 10.00 am - 12.00 pm

Instruction : The symbols used in the paper have their usual meaning, unless specified.

Q: 1. Answer the following by choosing correct answers from given choices. 10

[1] The discrete topology on a non-empty set X is ______ its indiscrete topology
[A] coarser than [B] finer than [C] not comparable with [D] none

[2] The discrete topology of a nonempty set is ______ any other topology on that set.
[A] coarser than [B] finer than [C] non-comparable with [D] none

[3] Any topology on a non-empty set is ______ the discrete topology on that set
[A] coarser than [B] finer than [C] non-comparable with [D] none

[4] If A is a dense subset of a topological space $(X, \mathcal{T})$ then
[A] $A' = X$ [B] $A = X$ [C] $A^- = X$ [D] none

[5] Minimum number of open as well as closed subsets in a topological space is
[A] 0 [B] 1 [C] 2 [D] 3

[6] If A is a closed set in a topological space then
[A] $A \subset A'$ [B] $A^- \neq A$ [C] $A = A'$ [D] $A' \subset A$

[7] Every non-empty and bounded above subset of R posseses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

[8] If I is an open interval then the subspace $(I, \mathcal{U}_I)$ and $(R, \mathcal{U})$
[A] both are compact [B] are homeomorphic
[C] both are bounded [D] none

[9] In a T_1 space the complement of every singleton set is
[A] closed [B] open [C] closed and open both [D] neither open not closed

[10] In the relativized topology of $\mathcal{U}$-topology of $\mathbb{R}$ the subset $[1, 10]$ is
[A] Compact [B] Disconneceted [C] Unbounded [D] none

Q: 2. In the following, depending on the type of question either fill in the blank or answer whether a statement is true false 08

[1] If (X, T) is a topological space then every subset of X is a closed set(True/False?)

[2] A closed set cannot be a neighbourhood of any of its points? (True/False?)

[3] In $(R, \mathcal{U})$ the set of irrationals is dense. (True/False?)

[4] A set is always a subset of its closure (True/False?)

[5] Let f be a function from a topological space (X, T) into another topological space (Y, V) such that $A \subset X$ is connected but $f(A)$ is not connected in Y. Then f is not continuous on X. (True/False).

[6] In a $(R, \mathcal{U})$ the subspaces $(0,1)$ ans $(0,100)$ are homeomorphic (True/False).

[7] A T_1 space is a Housdorff space. (true/False?)

[8] T_1 Space and Regular spaces are same.(True/False?)

Q: 3. Answer ANY TEN of the following. **20**

[1] If $X = \{a, b, c\}$ then find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ for X such that $\mathcal{T}_1 \subsetneq \mathcal{T}_2 \subsetneq \mathcal{T}_3$

[2] Define : (i) Usual Topology of $\mathbb{R}$ (ii) Coarser Topology

[3] Define : (i) Closed Set (ii) Non-comparable topologies

[4] Define : Homeomorphism

[5] For any topologies $\mathcal{T}$ and Ψ of $\mathbb{R}$ show that the mapping $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 2, \forall x \in \mathbb{R}$, is $\mathcal{T}$-Ψ continuous

[6] Find $\mathcal{U}$-closures of the sets $\mathbb{R}$ and $\emptyset$.

[7] Define : Hausdorff Space

[8] For $X = \{0, 1, 2, 3, 4, 5\}$ consider the topology $\mathcal{T} = \{X, \emptyset, \{0, 1, 2\}, \{3, 4, 5\}\}$. Is (X, T) connected?

[9] State the Least Upper Bound property of R

[10] Define : (i) T_3-space (ii) Metric Topology

[11] Prove that the space $(R, \mathcal{U})$ is a T_2-space.

[12] Prove that every metric space is a Hausdorff space

Q: 4. Attempt any FOUR of the following. **32**

[1] Let J be the set of all integers and $\mathcal{J}$ be a collection of subsets G of J where $G \in \mathcal{J}$ whenever $G = \emptyset$ or $G \neq \emptyset$ and $p, p \pm 2, p \pm 4, ..., p \pm 2n, ...$ belong to G whenever $p \in G$.Prove that $\mathcal{J}$ is a topology for J

[2] If $(X, \mathcal{T})$ is a topological space and $\{F_\alpha / \alpha \in \Lambda\}$ is any collection of $\mathcal{T}$-closed subsets of X then prove that $\bigcap\{F_\alpha / \alpha \in \Lambda\}$ is a $\mathcal{T}$-closed set

[3] Let $(X, \mathcal{T})$ and (Y, Ψ) be topological spaces and f be a mapping from X into Y. Prove that if $f(A^-) \subset f(A)^-$ for $A \subset X$, then the inverse image of f of every Ψ-closed set is $\mathcal{T}$-closed set.

[4] Let $(X, \mathcal{T})$ be a topological space and $A \subset X$. Prove the following

(i) $Int(A) \subset A$

(ii) $Int(A)$ is a $\mathcal{T}$-open set

(iii) A is $\mathcal{T}$-open iff $Int(A) = A$

(iv) $Int(A)$ is the largest open subset of A

[5] Prove that the space $(R, \mathcal{U})$ is connected.

[6] Prove that if $(X, \mathcal{T})$ is disconnected then there is a nonempty proper subset of X that is both $\mathcal{T}$-open and $\mathcal{T}$-closed.

[7] Prove that every T_3-space is a T_2-space

[8] If Y is a bounded and $\mathcal{U}$-closed subset of R, then prove that $(Y, \mathcal{U}_Y)$ is compact.

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