Seat No.: ______

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SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester 5) Examination – 2022

US05CPHY22: Mathematical Methods

Date: 11/11/2022 Friday

Time: 10:00 am to 1:00 pm

Total: 70 Marks

NOTE:

1. Figure to the right indicate full marks of the questions.
2. The symbols have their usual meaning.

Q-1 Multiple Choice Questions [10]

(1) In equation: $ds^2 = h_1^2du^2 + h_2^2dv^2 + h_3^2dw^2$; h_1, h_2 & h_3 are called ____.
- (a) Miller indices
- (b) Metrical coefficients
- (c) Complex numbers
- (d) Binary numbers

(2) ∇^2 is called ______.
- (a) Curl
- (b) Gradient
- (c) Laplacian
- (d) Divergence

(3) $\Gamma(n+1) =$ _______.
- (a) $n\Gamma n$
- (b) $n\Gamma(n+1)$
- (c) $(n-1)\Gamma(n-1)$
- (d) $n\Gamma(n-1)$

(4) For Bessel's equation, condition ______ is true.
- (a) $k = n$ or $k = -n$
- (b) $k = 1$ or $k = -1$
- (c) $k = n$ or $k = n - 1$
- (d) $k = n$ or $k = -n - 1$

(5) For Bessel's polynomial, the generating function is given by______.
- (a) e^{2x-t^2}
- (b) e^{2tx-t^2}
- (c) $e^{\frac{x}{2}(t-\frac{1}{t})}$
- (d) $e^{\frac{x}{2}(t^2-1)}$

(6) For a Fourier series of a periodic function $f(t)$ in $[-\infty, \infty]$, coefficients $\beta_n =$____.
- (a) $(a_n + a_{-n})$
- (b) $i(a_n + a_{-n})$
- (c) $(a_n - a_{-n})$
- (d) $i(a_n - a_{-n})$

(7) The amount of heat ΔH crossing an element of surface ΔS in time Δt is given by
- (a) $\Delta H = K\,\Delta S \left|\frac{du}{dt}\right|$
- (b) $\Delta H = K\,\Delta S\,\Delta t \left|\frac{du}{dt}\right|$
- (c) $\Delta H = K\,\Delta t \left|\frac{du}{dt}\right|$
- (d) None of these

(8) $y = ax^2 + bx + c$ is the equation of_______.
- (a) Parabola
- (b) Hyperbola
- (c) Straight Line
- (d) Ellipse

(9) The forward difference operator Δ defined as ______.
- (a) $\Delta y_i = y_{i+1} - y_i$
- (b) $\Delta y_i = y_i - y_{i+1}$
- (c) $\Delta y_i = y_{i-1} - y_i$
- (d) $\Delta y_i = y_i - y_{i-1}$

(10) In the Simpson's $\frac{1}{3}^{rd}$ rule, we have to use two subintervals of _____ width.
- (a) Gradually increase
- (b) Equal
- (c) Gradually decrease
- (d) Very large

Q-2 Short Answer Questions (Attempt TEN out of TWELVE) [20]

(1) Define beta function.

(2) Show that $\beta(m,n) = \beta(n,m)$.

(3) Write Laplacian in terms of orthogonal curvilinear co-ordinates.

(4) Show that $P_n(-\mu) = (-1)^n P_n(\mu)$.

(5) Show that $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$.

(P. T. O.)

(6) Write Hermite's differential equation.
(7) Write three-dimensional diffusion equation.
(8) Write cosine series for $f(x)$ when $0 \leq x \leq \pi$. (Note: derivation is not required)
(9) Find a_0 for $f(x) = x + x^2$ in the interval $-\pi \ll x \ll \pi$.
(10) Write the principle of least squares.
(11) Define interpolation and extrapolation.
(12) Derive an equivalent equation of a straight line for $y = ae^{bx}$.

Q-3 (A) Prove that the product of sets of two triads of mutually orthogonal vectors are reciprocal to each other. [06]

(B) If $u = x + 5,\ v = 2y - 4,\ w = 3z + 1$ show that u, v, w are orthogonal and find ds^2 and metrical coefficients h_1, h_2, h_3. [04]

OR

(A) Define orthogonal curvilinear co-ordinates and derive expression of curl in terms of orthogonal curvilinear coordinates. [06]

(B) If $x = u\,v \cos w, y = u\,v \sin w, z = \frac{1}{2}(u^2 - v^2)$, find h_1, h_2, h_3 and show that $ds^2 = (u^2 + v^2)(du^2 + dv^2) + u^2v^2dw^2$ [04]

Q-4 (A) State and derive Rodrigue's formula. [06]

(B) Show that $P_n(\mu)$ is the coefficient of h^n in $(1 - 2\mu h + h^2)^{-1/2}$. [04]

OR

(A) Derive the series solution of Bessel's differential equation in the form of ascending power of x. [06]

(B) Using equation: $H_n(x) = e^{x^2}(-1)^n \frac{d^n\, e^{-x^2}}{dx^n}$, find out $H_0(x)$ and $H_1(x)$. [04]

Q-5 (A) Define Fourier series and derive the expression of Fourier series for a periodic function $f(x)$ in the interval $(-\pi, \pi)$. [06]

(B) Derive one dimensional diffusion equation for one dimensional flow of electricity in a long-insulated cable. [04]

OR

(A) Obtain Fourier series for the expansion of $f(x) = x \sin x$ in the interval $-\pi \ll x \ll \pi$. [06]

(B) Derive one dimensional wave equation by considering a flexible string of length l tightly stretched between two points $x = 0$ and $x = l$ on $X - axis$. [04]

Q-6 (A) Evaluate $f(2.5)$ using Lagrange's interpolating polynomial from the following data. [06]

x	1.5	3.2	4
$y = f(x)$	3	6.4	8

(B) Derive Lagrange's interpolation formula. [04]

OR

(A) Find the approximate value of [06]

$$y = \int_0^{\pi} \cos x\, dx$$

using Simpson's 1/3 rule by dividing the range of integration into six equal parts. What is the analytical value of the above integral?

(B) Derive Simpson's 1/3rd rule in composite form. [04]

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