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SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester 5) Examination – 2022

US05CMTH24 – Metric Spaces and Topological Spaces

Date: 14/11/2022

Time: 10:00 A.M. To 01:00 P.M. Total: 70 Marks

Q-1. Multiple Choice Questions [10]

(1) Let $d: R \times R \to R$. Which of the following is not metric on R.

A. $d(x,y) = |x^2 - y^2|$
B. $d(x,y) = |x - y|$
C. $d(x,y) = |y - x|$
D. $d(x,y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$

(2) Let $f: M_1 \to M_2$. (M_1, ρ_1) & (M_2, ρ_2) are metric spaces. if $f^{-1}(F)$ is closed for all closed subset F of M_2 then

A. f is one-one
B. f is surjective
C. f is monotonic.
D. f is continuous

(3) For which of the following set A and A^C both are dense in R?

A. N
B. Z
C. R
D. Q

(4) For R with discrete metric, which of the following subset is totally bounded?

A. [3,5]
B. (0,5)
C. {1,2,3}
D. $\left\{\frac{1}{n} : n \in N\right\}$

(5) Which of the following set is u –open set?

A. $\{n : n \in Z\}$
B. $\{0, \pm 1, \pm 2\}$
C. $(-1,1)$
D. $[-1,1]$

(6) Let $\tau_1 = \{\phi, \{a,d\}, \{a,b,c,d\}\}, \tau_2 = \{\phi, \{a\}, \{d\}, \{a,b,c,d\}\}$. Then,

A. τ_1 is finer than τ_2
B. τ_1 and τ_2 are not comparable
C. τ_2 is finer than τ_1
D. None

(7) A is τ –open set iff

A. $\bar{A} = A$
B. $X - A = A$
C. $\bar{\bar{A}} = A$
D. $int(A) = A$

(8) If there is a non-empty proper subset of X which is both τ –open and τ –closed then

A. X is compact
B. X is connected
C. X is disconnected
D. None

(9) What is the greatest lower bound of the set $A = \{x \in R: -3 < x < 3\}$?

A. -3
B. 3
C. 0
D. 9

(10) Which of the following set is not compact in usual topology?

A. $[0,1]$
B. $[-10,10] \cup (-1,1)$
C. $(-\infty, \infty)$
D. $\{0\}$

P.T.O.

Q-2. Short Answer Questions (Attempt any TEN) [20]

(1) Define a Metric and give any one example of metric defined on R^2.
(2) Prove that if ρ is a metric on X, then 5ρ is also a metric on set X.
(3) Define: Open Ball in Metric Space.
(4) What is the distance between $(1,1)$ and $(1,-1)$ in R^2 with metric $d(x,y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$?
(5) Give any three topologies on $X = \{a, b, c, d\}$.
(6) Is the set $A = \left\{\frac{1}{n} : n \in N\right\}$ closed in R with usual metric? Justify.
(7) Prove that constant function is continuous in (R, u).
(8) Define: u-open set in Topological Space.
(9) Define: Interior point in Topological Space
(10) Define: Disconnected Space in Topological Space.
(11) Find set of all cluster points of R in discrete topology.
(12) Give any one open covering of R which has no finite sub-covering.

Q-3 (a) Let $\rho: R^n \times R^n \to R$ defined by $\rho(x,y) = \sqrt{\sum_{k=1}^{n}(x_k - y_k)^2}$. Where $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$. Then prove that ρ is metric on R^n. 05

(b) If G_1 and G_2 are open subsets of the metric space M, then prove that $G_1 \cup G_2$ is also open in M. 05

OR

(a) Let $\rho: R \times R \to R$ defined by $\rho(x,y) = |x - y|$. Then prove that ρ is metric on R. 05

(b) Prove that the real valued function f is continuous at $a \in R$ if and only if whenever $\{x_n\}$ is a sequence of real numbers converging to 'a' then the sequence $\{f(x_n)\}$ converges to $f(a)$. 05

Q-4 (a) Define Usual topology and prove that it possesses all the conditions to becoming a Topology. 05

(b) Prove that singleton sets and finite sets are closed in R with usual topology. 05

OR

(a) Let $\mathcal{G}$ be a family of subsets of R as described by 05

(i) $\Phi \in \mathcal{G}$

(ii) If $G \subset R$ & $G \neq \Phi$ then $G \in \mathcal{G}$ if for each $p \in G$ there is a set $H = \{x \in R : a \leq x < b\}$ such that $p \in H \subset G$.

Then prove that $\mathcal{G}$ is an unusual non-trivial topology of R.

(b) Prove that arbitrary intersection of closed sets is also closed in Topological space. 05

Q-5 (a) Let (X, τ) be a topological space and let A be a subset of X. Prove that A is τ-open if and only if A contains a τ-neighbourhood of each of its points. 05

(b) Let (X, τ) be a topological space and let A be a subset of X. A' is the set of all cluster points of A then prove that $A \cup A'$ is τ-closed. 05

OR

(a) Prove that $Int(A)$ is largest open subset of A. 05

(b) Let $f: (R, \tau) \to (R, \psi)$ defined by $f(x) = 2, \forall x \in R$. Then prove that f is $\tau - \psi$ continuous. 05

Q-6 (a) Prove that continuous image of connected space is connected. 05

(b) Define relative topology and show that relative topology satisfies all the conditions for becoming a topological space. 05

OR

(a) Let (X, τ) be a topological space and let $Y \subset X$. Prove that if the subspace (Y, τ_Y) is connected then the subspace $(\bar{Y}, \tau_{\bar{Y}})$ is also connected. 05

(b) Prove that R with usual topology is not compact space. 05

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