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Sardar Patel University, Vallabh Vidyanagar

B.Sc. Examinations : Sem 5 Subject : Mathematics Max. Marks :70

Date: 11/11/2022 US05CMTH22 [Theory of Real Functions] Time : 10.00 to 1.00 a.m.

Q.1 Choose the correct option for each of the following. [10]

(1) If $\lim_{x \to a} f(x)$ exists but f(a) is not defined then f is said to have discontinuity of ....

(a) first type (b) second type (c) removable type (d) first type from left

(2) A function is said to be continuous in a region if it is continuous at .... of the given region.

(a) Only one Point (b) Every Point (c) Some Point (d) Nowhere

(3) $f(x) = |x|$ is .... at $x = 0$.

(a) discontinuous (b) differentiable (c) not differentiable (d) None of these

(4) If $f(x) \le f(y), \forall x \le y$ then the function f is said to be ....

(a) increasing (b) decreasing (c) strictly increasing (d) strictly decreasing

(5) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots = \ldots\ldots$

(a) e^x (b) $\log(1+x)$ (c) $sinx$ (d) $cosx$

(6) Which of the following is Cauchy's form of reminder in the Taylor's theorem

(a) $R_n = \frac{h^n[1-\theta]^{n-p}}{p[(n-1)!]} f^{(n)}(a+\theta h)$ (b) $R_n = \frac{h^n[1-\theta]^{n-1}}{[(n-1)!]} f^{(n)}(a+\theta h)$

(c) $R_n = \frac{h^n}{[n!]} f^{(n)}(a+\theta h)$ (d) None of these

(7) For $f(x,y) = x^3 - xe^y$ then the value of $f_x(1,0)$ is ....

(a) 0 (b) 1 (c) 2 (d) 3

(8) $\lim_{(x,y)\to(4,\pi)} x^2 \sin\frac{y}{x} = \ldots\ldots$

(a) 0 (b) 8 (c) -8 (d) $8\sqrt{2}$

(9) The extreme value of $f(a,b)$ is called maximum if $f(x,y) - f(a,b)$ is ....

(a) Alternate +ve & -ve (b) Positive (c) Negative (d) None of these

(10) A stationary point is called saddle point of function f if it is .... point

(a) Extreme (b) Non extreme (c) Stationary (d) None of these

Q.2 Attempt any TEN: [20]

(1) Define: A continuous function at a point.

(2) Prove that $\lim_{x\to 0} x.\ \sin\frac{1}{x} = 0$.

(3) Prove that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on (0,1]

(4) State Rolle's theorem.

(5) If a function f satisfies the conditions of Lagrange's Mean value theorem and $f'(x) = 0, \forall x \in (a,b)$ then prove that $f(x)$ is constant on $[a,b]$.

(6) State Maclaurin's theorem.

(7) Evaluate : $\lim_{(x,y)\to(0,0)} \frac{x\sin(x^2+y^2)}{(x^2+y^2)}$.

(8) Define : Repeated limits of a function of two variables.

(9) Using definition of partial derivatives find f_x and f_y of

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & if (x,y) \ne (0,0) \\ 0, & if (x,y) = (0,0) \end{cases}$$

(10) Define : Extreme Value.

(11) State necessary condition for a $f(x,y)$ to have an extreme value at a point (a,b).

(12) Expand $x^2y + 3y - 2$ in powers of $x - 1$ and $y + 2$

(P. T. O.)

Q.3 (a) Show that a continuous function on a closed interval [a,b] attains its bounds at least once in [a,b] [5]

(b) If f and g are two functions defined on some neighbourhood of c such that $\lim_{x \to c} f(x) = l$ [5]

and $\lim_{x \to c} g(x) = m, m \neq 0$ then prove that $\lim_{x \to c} (\frac{f}{g})(x) = \frac{l}{m}$.

OR

Q.3 (a) Prove that limit of a function is unique if exists. [5]

(b) If f is continuous on $[a, b]$ then prove that f is uniformly continuous on $[a, b]$. [5]

Q.4 (a) State and Prove Darboux's theorem. [5]

(b) Prove that $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, \forall x > 0.$ [5]

OR

Q.4 (a) State and Prove Lagrange's mean value theorem. [5]

(b) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}, 0 < x < \frac{\pi}{2}$. [5]

Q.5 (a) Define : Limit of a function of two variables and by using the definition of limit prove that [5]

$\lim_{(x,y)\to(1,2)} (x^2 + 2y) = 5$.

(b) Prove that $\lim_{(x,y)\to(0,0)} xy\frac{(x^2-y^2)}{(x^2+y^2)} = 0.$ [5]

OR

Q.5 (a) Prove that $f(xy, z - 2x) = 0$ satisfies the equation $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$ under suitable condition. [5]

State these conditionS .

(b) Prove that for a given function $f(x,y) = \begin{cases} x\sin\left(\frac{1}{y}\right) + y\sin\left(\frac{1}{x}\right), & xy \neq 0 \\ 0, & xy = 0 \end{cases}$ [5]

Limit exists at the origin,but the repeated limits do not.

Q.6 (a) State and Prove Taylor's theorem for two variables. [5]

(b) Investigate the maxima and minima of the function $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$. [5]

OR

(a) Prove that the first four terms of the Maclaurin's expansion of $e^{ax}\cos by$ are [5]

$1 + ax + \frac{a^2x^2 - b^2y^2}{2!} + \frac{a^3x^3 - 3ab^2xy^2}{3!}$.

(b) Prove that $2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0,0). [5]

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