[36] SEAT No.__________ No. of Printed Pages: 02

Sardar Patel University, Vallabh Vidyanagar

B.Sc. Examinations : Sem 5 Subject : Mathematics Max. Marks :70

Date: 10/11/2022 Thursday US05CMTH21 [Real Analysis] Time : 10.00 to 1.00

Q.1 Choose the correct option for each of the following. [10]

(1) The field which does not have the least upper bound property is ....

(a) Q (b) Z (c) N (d) None of these

(2) The greatest number of a set ,if exists is ....

(a) the supremum of the set (b) the infimum of the set (c) not unique (d) None of these

(3) The smallest number of $\left\{1+\frac{1}{n} / n \in N\right\}$ is ....

(a) 0 (b) 1 (c) 2 (d) None of these

(4) Every closed interval in R is ....set

(a) an open (b) a closed (c) open and closed (d) None of these

(5) The derived set of A ={-2,-1,0,1,2,3} is ....

(a) Ø (b) R (c) A (d) Z

(6) The closure of Q i.e $\tilde{Q}$ is ....

(a) N (b) Q (c) Ø (d) R

(7) The Range of sequence is always ....

(a) empty (b) infinite (c) non- empty (d) None of these

(8) Sequence $\{(-1)^n / n \in N\}$ is ....

(a) convergent (b) divergent (c) oscillates finitely (d) None of these

(9) A positive term series $\sum \frac{1}{n^p}$ is convergent iff ....

(a) p = 1 (b) $0 < p < 1$ (c) $p > 1$ (d) $p < 0$

(10) A series $\sum u_n$ is convergent then $\lim_{n\to\infty} u_n$

(a) $\neq 0$ (b) $= 0$ (c) $= 1$ (d) does not exists

Q.2 Attempt any TEN: [20]

(1) Define : An Ordered Field.

(2) Prove that the least upper bound of a set S is unique ,if it exist.

(3) Find the g.l.b and l.u.b of $\left\{\frac{1}{m}+\frac{1}{n} / m,n \in N\right\}$ if they exist.

(4) Prove that every open intreval is an open set.

(5) Define: A closure of a set.

(6) Define: A limit point of a set.

(P.T.O)

(7) Define: A convergent sequence .

(8) Prove that every convergent sequence is bounded.

(9) Prove that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

(10) Define : An infinite series

(11) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.

(12) Investigate the behaviour of the series whose n^{th} term is $\frac{n}{n+1}$.

Q.3 (a) State and Prove the Archimedean property of R . [5]

(b) Prove that $\sqrt{6}$ is an irrational number. [5]

OR

Q.3 (a) Prove that the set of all rationals Q is not an order complete field. [5]

(b) If $x, y \in R$ then prove that $|x+y| \leq |x| + |y|$. Also prove that $||x| - |y|| \leq |x-y|$. [5]

Q.4 (a) Prove that a set is closed iff its complement is open . [5]

(b) Prove that the arbitrary union of open sets is open . [5]

OR

Q.4 (a) If S and T are sets of real numbers then prove the following [5]

(i) $S \subset T \Rightarrow S' \subset T'$ (ii) $(S \cup T)' = S' \cup T'$

(b) Let $S \subset R$ then prove that the interior of S is the largest open subset of S. [5]

Q.5 (a) State and Prove Bolzano-Weierstrass theorem for sequence. [5]

(b) If $\{a_n\}$ and $\{b_n\}$ are two sequences such that (i) $a_n \leq b_n, \forall n \in N$ [5]

(ii) $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ then prove that $a \leq b$.

OR

(a) State and Prove Cauchy's first theorem on limits. [5]

(b) Prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$. [5]

Q.6 (a) State and Prove comparision test of first type. [5]

(b) Prove that the positive term geometric series $1 + r + r^2 + \cdots$ converges for $r < 1$ [5]

and diverges to ∞ for $r \geq 1$.

OR

(a) State and Prove Cauchy's Root Test. [5]

(b) Prove that the series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \ldots\ldots$ converges. [5]

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