SEAT No.__________ No. of Printed Pages: 2

[A-2]

**SARDAR PATEL UNIVERSITY**
**SEM IV , MATHEMATICS**
**Calculus and Algebra-II(US04EMTH05)**

**Date: 12-10-2022** **Time: 12-30 to 2-30**

**1. Answer the following by selecting correct choice from the options :** **[10]**

(1) If $x^3 + y^3$, then $f_{xy} =$______
(a) $2x$ (b) $0$
(c) $2y$ (d) $2$

(2) If $AC - B^2 > 0$ and $A < 0$ then ____
(a) local minimum and local maximum of $f$ do not exist. (b) can't say anything
(c) $f(a,b)$ is a local maximum of $f$ (d) $f(a,b)$ is a local minimum of $f$

(3) If $f(x,y)$ is sufficiently many times differentiable in same neighborhood of $(a,b)$ then $f_{xy}(a,b) =$________
(a) $A$ (b) $B$
(c) $C$ (d) none

(4) $grad(\emptyset + f) =$____.
(a) $gradf$ (b) $grad\emptyset + gradf$
(c) $grad\emptyset$ (d) 0

(5) If $f(x,y,z) = x + 2y$, then $\overline{\nabla} f =$______
(a) $3\bar{\imath} + 3\bar{\jmath}$ (b) $3\bar{\imath} + 6\bar{\jmath}$
(c) $\bar{\imath} + 2\bar{\jmath}$ (d) $x\bar{\imath} + y\bar{\jmath}$

(6) Curl of $V$ is denoted by ______
(a) $div\ V$ (b) $\overline{\nabla} \times V$
(c) $\overline{\nabla} \cdot V$ (d) none

(7) $\overline{\nabla} \times (\overline{\nabla} f) =$_______
(a) 0 (b) $\bar{0}$
(c) 1 (d) $none$

(8) If $a \in B$, where $B$ is Boolean algebra, $a + a =$________.
(a) 1 (b) 0
(c) $a'$ (d) $a$

(9) In Boolean algebra, $1' =$______variable
(a) $1'$ (b) $1$
(c) $0$ (d) none

(10) For every $a, b \in B, B$ is Boolean algebra, $a \cdot (a + b) =$______
(a) $a$ (b) 0
(c) $1$ (d) $b$

**2. Do as directed.** **(8)**

1) True/False: A stationary point which is not an extreme point is called saddle point.
2) Fill in the blank: $D_2 f =$______$\left(\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}\right)$.
3) True/False: The value of first derivative at point of Minima is not 0.



(P.T.O.)

4) Fill in the blank: Div(curl $f$) = _____(0/doesn't exist).
5) Fill in the blank: In Boolean Algebra, $x + x =$ _____$(x/0)$.
6) Fill in the blank: In Boolean Algebra, $(A \cap B)' =$ _____
7) True/False: In Boolean Algebra, $a \cdot (b + c) = a \cdot b + a \cdot c$
8) True/False: $(ab)' = a' \cdot b'$

**3. Answer any TEN of the following.** **[20]**

1) Define Stationary point.
2) Find Curl of $x\bar{i} + y\bar{j} + z\bar{k}$.
3) Define local maxima.
4) If $f(x, y, z) = x^2 - 2y^2 + 3z^2$, find $\overline{\nabla} f$ at $(3, 4, 5)$.
5) Define: Gradient of a scalar field.
6) Prove that $\overline{\nabla}(fg) = f\,\overline{\nabla} g + g\overline{\nabla} f$
7) If $\bar{V} = xy\hat{i} + x^2\hat{j}$ then find Curl $\bar{V}$.
8) Show that $\overline{\nabla} f(r) = f'(r)\nabla r$
9) Prove that $\overline{\nabla} \cdot (\overline{v_1} - \overline{v_2}) = \overline{\nabla} \cdot \overline{v_1} - \overline{\nabla} \cdot \overline{v_2}$
10) Draw network represented by the function $(x + y')(x' + y)$.
11) State and prove De-Morgan's law for Boolean Algebra.
12) If $a + x = b + x$ & $a + x' = b + x'$ then prove that $a = b$.

**4. Attempt any FOUR** **(32)**

1) Show that $(y - x)^4 + (x - 2)^4$ has minimum at (2, 2).
2) Show that the function $2x^4 - 3x^2y + y^2$ has neither maximum nor minimum at (0, 0).
3) Prove that $f(x, y) = tan^{-1}\left(\frac{y}{x}\right)$ is harmonic function.
4) Find direction derivative of $f(x, y, z) = 2x^2 + 3y^2$ at point $(2,1,3)$ in the direction of $\bar{a} = \bar{i} - 2\bar{k}$.
5) If $f(x, y) = log(x^2 + y^2)$ then verify $\nabla^2 f = 0$
6) Prove that $\nabla(r^n\bar{r}) = (n + 3)r^n$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ , $= |\bar{r}|$ .
7) Prove that In every Boolean algebra B each of the binary operations (+) and (·) are associative.
8) Simplify following function and draw network represented by the function $x + xy'$.

— x —