SEAT No.__________ No. of Printed Pages: 2

[A-12(A)]

SARDAR PATEL UNIVERSITY (B.Sc. Sem.4 Examination (NC))

MATHEMATICS - US04EMTH05 - Calculus and Algebra - 2

Date: 28-9-2022, Wednesday

Time: 12:30 TO 02:30 p.m. **Maximum Marks: 70**

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options. **[10]**

1 If _____ then (a, b) is stationary point.

(A) $f_x(a,b) = 0$ (B) $f_y(a,b) = 0$ (C) $f_x(a,b) = f_y(a,b) = 0$ (D) None

2 $f(x,y) = x^2 + 2y$ then AC-B^2_______

(A) > 0 & $A > 0$ (B) < 0 & $A > 0$ (C) $= 0$ & $A > 0$ (D) None

3 Saddle point is ________ a local extremum of the function.

(A) not (B) always (C) sometimes (D) none of these

4 $\nabla f(r)$ =______, for $\bar{r} = xi + yj + zk$.

(A) $\nabla f'(r)\,\nabla r$ (B) $\nabla f(r)\,\nabla r$ (C) $\nabla f(r)\,\nabla r'$ (D) none of these

5 $\nabla(f + g)$ = _______

(A) $\nabla f - \nabla g$ (B) $\nabla f + \nabla g$ (C) $f(\nabla g) + g(\nabla f)$ (D) none of these

6 curl $\bar{v}$ =_______, for $\bar{v} = xi + 2yj + 3zk$.

(A) 0 (B) 1 (C) 2 (D) 3

7 If $\bar{r} = xi + 3yj$ then $\nabla \cdot \bar{r}$= ________

(A) 1 (B) 2 (C) 3 (D) 4

8 In Boolean algebra, a+a·b=_______

(A) a (B) b (C) ab (D) a+b

9 In Boolean algebra, a·a=________

(A) a^2 (B) a (C) 1 (D) 0

10 In Boolean algebra, a·0=________

(A) a^2 (B) a (C) 1 (D) 0

Q.2 Answer the given statement is TRUE or FALSE **[08]**

1 There always exist stationary points of function.

2 Stationary point may not become extreme point.

3 Gradient of vector is a scalar quantity.

4 $\nabla(f - g) = f(\nabla g) - g(\nabla f)$

5 Divergence of vector is a scalar quantity.

6 Curl of vector is a vector quantity.

7 In Boolean algebra, $a + a'b = a + b$

8 In Boolean algebra, $a \cdot a' = 0$

[P.T.O.]

Q.3 Answer ANY TEN of the following. **[20]**

1. State necessary condition for a maximum and minimum of a differentiable function.
2. Define: Saddle point
3. What is extreme value of a function?
4. Define: Directional derivative of scalar point function
5. Show that $\nabla(fg) = f(\nabla g) + g(\nabla f)$
6. Show that $\nabla(f^n) = nf^{n-1}\nabla f$.
7. Show that $\nabla(\nabla f) = \nabla^2 f$
8. In usual notation prove that $\nabla \times (\bar{v} + \bar{u}) = \nabla \times \bar{v} + \nabla \times \bar{u}$
9. In usual notation prove that $\nabla \cdot (\nabla \times \bar{v}) = 0$.
10. Draw the network represented by $z(x' + yz + y')$.
11. For all a in Boolean algebra (B), prove that $(a')' = a$.
12. In a Boolean algebra B, prove that $a(a' + b) = ab$.

Q-4 Answer ANY FOUR of the following. **(32)**

1. Investigate the maxima and minima of the function
$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$
2. Show that $y^2 + x^2y + x^4$ has a minimum at (0, 0).
3. Show that the function $f(x, y) = tan^{-1}\left(\frac{y}{x}\right)$ is a harmonic function.
4. Find unit normal vector to the surface $z^2 = 4(x^2 + y^2)$ at $(1, 2, 3)$.
5. Verify $\nabla(f\nabla g) = f\nabla^2 g + \nabla f\, \nabla g$ for $f = x + y + z$, $g = xyz$.
6. Find curl $\bar{v}$ for $\bar{v} = \frac{\bar{r}}{|\bar{r}|^3}$, where $\bar{r} = xi + yj + zk$.
7. For all a in Boolean algebra B, inverse is unique.
8. Find Boolean function for the following circuit. Simplify it and draw simplified circuit.

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