



SEAT No. _____

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[2/A-I]

SARDAR PATEL UNIVERSITY

B.Sc. Semester-IV Examination

Saturday, 15th October, 2022

Time:-12:30 P.M. to 02:30 P.M. Paper Code:- US04CSTA02 M.Marks:70

Subject:-Probability Distribution

Note:- Simple/Scientific calculators are allowed. Statistical Table is allowed.

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Q.1. Multiple Choice Questions: -

- 1 The coefficient of skewness for a binomial distribution will be zero if _____.
(a) $p < \frac{1}{2}$ (b) $p > \frac{1}{2}$ (c) $p = \frac{1}{2}$ (d) $p < q$
- 2 A student randomly guesses at each of 8 multiple choice questions. Each question has 5 possible choices. The probability that the student gets exactly 4 correct is _____.
(a) 0.115 (b) 0.227 (c) 0.046 (d) None of these
- 3 If $M_X(t) = (\frac{1}{3} + \frac{2}{3} \exp(t))^9$ is the m.g.f. of X then $\mu'_1 =$ _____.
(a) 6 (b) 5 (c) 4 (d) 3
- 4 If $X \sim N(\mu, \sigma^2)$ then $(\frac{x-\mu}{\sigma})$ has _____.
(a) $N(0, 1)$ (b) $N(0, \sigma^2)$ (c) χ^2_1 (d) χ^2_n
- 5 The _____ distribution has all odd order moments zero.
(a) Chi-square (b) Normal (c) Student's t (d) both(b) &(c)
- 6 If X and Y are two independent exponential variates with mean θ each, then $Z = X+Y$ follows _____ distribution.
(a) $G(2, \theta)$ (b) $G(2, 2\theta)$ (c) $G(1, \theta)$ (d) none of these
- 7 If $S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$ is the sample variance of a random sample from $N(\mu, \sigma^2)$ then $\frac{(n-1)S^2}{\sigma^2} \sim$ _____.
(a) χ^2_n (b) χ^2_{n-1} (c) χ^2_1 (d) none of these
- 8 If $X \sim b(n, p)$ distribution then as $n \rightarrow \infty$, $(\frac{x-np}{\sqrt{npq}}) \sim$ _____.
(a) Standard normal (b) Poisson (c) Binomial (d) none of these
- 9 _____ is the Median of the standard normal distribution.
(a) 0 (b) 1 (c) 2 (d) None of these
- 10 If $X \sim N(0,1)$ and $Y \sim \chi^2_r$ variates and both are independent then $\frac{X}{\sqrt{Y/r}} \sim$ _____ distribution.
(a) $F(m, n)$ (b) $F(r, 1)$ (c) t_r (d) χ^2_r

[08]

Q.2 Fill in the blanks: -

- 1 The area under the normal curve between $z=0$ and $z=1$ is _____ the area under the normal curve between $z=-1$ and $z=0$.
- 2 If $X \sim F(m,n)$ distribution then the distribution of $\frac{1}{X}$ is _____.
- 3 The mean for the Gamma $G(\alpha,1)$ distribution is _____.
- 4 The area under $P(-2.25 < Z < 2.25)$ is _____.
- 5 State whether the statement is True or False.
The mean and variance of the Binomial distribution are equal.

- 6 The shape of the Normal distribution is not symmetrical.
 7 If X_1 and X_2 are two independent $N(1,2)$ and $N(2,2)$ variate then the distribution of $Y=X_1+X_2$ follows $N(3,4)$ distributions.
 8 Let $X \sim U(2, a)$ variate with mean 5. The value of a is 5.

Q.3. Short Questions: - (Attempt any Ten)

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- 1 The mean and standard deviation of binomial distribution are 9 and 2 respectively. Find its parameter.
 2 If $f(x) = \frac{1}{15}, x=1,2,\dots,15$
 $= 0$, otherwise is the p.m.f. of X . Find $E(X)$ and $E(2X+5)$.
 3 Define Binomial distribution. State the condition under which Binomial distribution tends to Poisson distribution.
 4 If X is uniformly distributed with mean 1 and variance $4/3$, find $P(X>0)$.
 5 The length X of a component produced by a machine is a r.v. having the p.d.f,
 $f(x) = k(1-x), 0 < x < 1$
 $= 0$, otherwise
 Determine the value of k .
 6 If $M_x(t) = (1 - 2t)^{-1}$, is the m.g.f. of a r.v. X then identify the distribution of X State its mean and variance.
 7 The m.g.f. of a r.v. X is $M(t) = (0.55 + 0.45e^t)^{20}$ Find approximate value of $P(3 < x \leq 8)$.
 8 If $M_x(t) = e^{25t(1+t)}$ is the m.g.f. of a continuous r.v. X then name the distribution and writes its p.d.f.
 9 If $X \sim P(64)$ then find $P(X=80)$. State clearly the result you have used to solve the same.
 10 If $X \sim N(0,1)$ distribution then finds $P(X^2 > 3.84)$. State clearly, the result you have used to solve the probability.
 11 Define F-distribution. Write the p.d.f. of $F(2,2)$ distribution.
 12 Define Chi-square distribution and Student t-distribution.

Q.4. Long Questions: - (Attempt any four)

[32]

- 1 An urn contains 4 white and 3 black balls. If two balls are drawn at random (i) with replacement (ii) without replacement, find the probability that the selected balls containing (a) exactly 1 (b) atleast 1, black balls.
 2 A test consists of 10 multiple choice questions, each with four possible answers, one of which is correct. To pass the test a student must get 60% or better on the test. If a student randomly guesses, what is the probability that the student will pass the test?
 3 State the m.g.f. of Normal distribution. Obtain its c.g.f. and hence β_1 and β_2 .
 4 For a given p.d.f.
 $f(x) = kx^2(1-x)^3, 0 < x < 1$
 $= 0$, otherwise
 Find (i) k (ii) $P(0.1 < X < 0.7)$ (iii) $E(X^r)$
 5 If X_1, X_2, \dots, X_n are independent r.v.'s with m.g.f. $M_{X_1}(t), M_{X_2}(t), \dots, M_{X_n}(t)$ then prove that the m.g.f. of $Y = \sum_{i=1}^n X_i$ is given by $M_Y(t) = \prod_{i=1}^n M_{X_i}(t)$.
 6 If X is $N(\mu, \sigma^2)$ variate. Show that $Y = aX + b$ is $N(a\mu + b, a^2 \sigma^2), a \neq 0$.

- 7 If X has $U[\theta_1 - \theta_2, \theta_1 + \theta_2]$. Find θ_1 and θ_2 so that the mean and variance of X are respectively equal to the mean and variance of $\chi^2_{(6)}$.
- 8 If X_1, X_2, \dots, X_{16} is a r.v. of size $n=16$ from $N(50, 100)$, determine
- (a) $P(796.2 \leq \sum_{i=1}^{16} (X_i - 16)^2 \leq 2630)$
- (b) $P(726.1 \leq \sum_{i=1}^{16} (X_i - \bar{X})^2 \leq 2500)$
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