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[A-4]

B.Sc. (IV-Semester) EXAMINATION 2022

Tuesday, 04th October

12:30pm-02:30pm

US04CMTH02-Mathematics

Differential Equations

Total Marks: 70

Note: Figures to the right indicates full marks of question.

Q: 1 Answer the following by selecting the correct answer from the given options: **[10]**

1. Orthogonal trajectories of the given curves intersect them at---------- angle.

   a. acute b. right c. obtuse d. none of these

2. One solution of $\frac{dx}{yz} = -\frac{dy}{xz} = \frac{dz}{xy}$ is given by ----------

   a. $x^2 + y^2 = c$ b. $x^2 + z^2 = c$ c. $x^2 - y^2 = c$ d. $xyz = c$

3. Integral curve of $xdx = -ydy = zdz$ is given by

   a. $x + y = c_1, z + y = c_2,$ b. $x^2 + z^2 = c$ c. $x^2 + y^2 + z^2 = c$ d. none of these

4. Eliminating the arbitrary function f from $z = f\left(\frac{x}{y}\right)$, we get----------

   a. $yp + xq = x$ b. $xp - yq = 0$ c. $xp + yq = 0$ d. $xp + yq = y$

5. The solution of the Pfaffian differential equation $ydx + xdy - zdz = 0$ is----------

   a. $x^2 + y^2 - z = c$ b. $2xy - z^2 = c$ c. $2xy - z = c$ d. $2x^2 + 2y^2 - z^2 = c$

6. There exist a relation $f(u, v) = 0$ between u and v not involving x and y explicitly if--------

   a. $\frac{\partial(u,v)}{\partial(x,y)} = 1$ b. $\frac{\partial(u,v)}{\partial(x,y)} = 0$ c. $\frac{\partial(u,v)}{\partial(x,x)} = 1$ d. . $\frac{\partial(u,v)}{\partial(x,x)} = 0$

7. A complete integral of the P.D.E. $p + q = pqz$ is

   a. $z = \sqrt{2x + a} + \sqrt{2y + b}$ b. $z = 2ax + b$ c. . $z = 2y + b$ d. $z = (2x + a)(2y + b)$

8. Order of the P.D.E. $\frac{\partial^3 z}{\partial x^3} + xy\left(\frac{\partial^2 z}{\partial y^2}\right)^4 + z^2\left(\frac{\partial z}{\partial y}\right)^5 = 0$ is

   a . 2 b. 3 c. 4 d. 5

9. Two equations $p = P(x, y),\ q = Q(x, y)$ are compatible if----------

   a . $\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$ b. $\frac{\partial P}{\partial z} = \frac{\partial Q}{\partial z}$ c. $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ d. none of these

10. In Charpit's method equation involving only p and q takes the form ----------form.

    a . $\frac{dp}{x} = \frac{dq}{y}$ b. $\frac{dp}{0} = \frac{dq}{0}$ c. $\frac{dp}{f_x} = \frac{dq}{f_y}$ d. $\frac{dx}{0} = \frac{dy}{0}$

Q: 2 Do as Directed: **[08]**

1. Integral curve $\frac{dx}{x} = \frac{dy}{z} = \frac{dz}{y}$ of is given by ----------
2. Two linearly independent solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ represents---------
3. True or False: In partial differential equation $\frac{\partial z}{\partial x}$ is denoted by q.
4. True or False: The equation $p + q = 0$ is a linear partial differential equation.
5. True or False: The equation $pq = 1$ is a non-linear partial differential equation.
6. True or False: Surface orthogonal to a given system of surfaces has a form $pP + qQ = R$
7. The general form of Clairut's equation is----------
8. Two partial differential equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible if----------

Q:3 Answer in brief of the following questions. ***(Any Ten)*** **[20]**

1. Find the integral curves of the equations $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z}$
2. Define: Orthogonal trajectories of a system of curves on a surface.
3. Solve: $\frac{dx}{y} = \frac{dy}{xz} = \frac{dz}{xy}$
4. Obtain partial differential equation for $z = (x + a)(y + b)$
5. Determine whether the equation $2xzdx + zdy - dz = 0$ is integrable or not?
6. Eliminate a and b from $z = ax^3 + by^3$
7. Find the differential equation of the surface, which orthogonal to the system of surfaces $x^2 + y^2 + z^2 = cz$.
8. Verify that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is the complete integral of $z^2(p^2 + q^2 + 1) = 1$
9. Find the particular integral of the P.D.E. $p + q = 1$ passes through the curve $x = 0, y^2 = z$.
10. Find Complete integral of the equation $pqz = p^2(xq + p^2) + q^2(yp + q^2)$
11. If $z = tx + yf(t) + g(t)$ then prove that $rt - s^2 = 0$.
12. Find C.F. of the equation $(D^2 - DD') = cosxcos2y$.

Q:4 Attempt any Four of the following. **[32]**

1. Solve: $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$
2. Obtain the differential equation of orthogonal trajectories for a given system of curves on the given surface.
3. Obtain the higher order P.D.E. for a function $z = f(x, y)$
4. Find the general solution of the given linear partial differential equation $x^2p + y^2q = (x + y)z$
5. Find the equation of the system of surfaces which cut orthogonally the system $x(x^2 + y^2 + z^2) = cy^2$.
6. Find the integral surface of the equation $x^2p + y^2q + x^2 = 0$ through the curve $xy = x + y, z = 1$.
7. State and prove required condition for compatibility of two partial differential equations.
8. Find complete integral of the equation $pqxy = z^2$.

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