SEAT No.________

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[A-3(B)]

SARDAR PATEL UNIVERSITY
B.Sc.(SEMESTER - IV) NC EXAMINATION - 2022
Monday, 3rd October 2022 MATHEMATICS: US04CMTH01
(Linear Algebra)

Time : 12:30 p.m. to 02:30 p.m. Maximum Marks : 70

Que.1 Fill in the blanks. 10

(1) If S is nonempty subset of vector space V_3 then (0,0,0) .... [S].

(a) $=$ (b) $\neq$ (c) $\in$ (d) $\notin$

(2) $[\phi] =$ ....

(a) 0 (b) $\{0\}$ (c) ϕ (d) V

(3) .... $\in [(1,2),(2,4)]$ in V_2 .

(a) $(0,1)$ (b) $(10,5)$ (c) $(8,4)$ (d) $(3,6)$

(4) $\{(2,0,0)$, $(0,3,0)$, ...$\}$ is LI set .

(a) $(0,0,1)$ (b) $(0,6,0)$ (c) $(2,3,0)$ (d) $(4,6,0)$

(5) Any set containing zero vector is .... set .

(a) LI (b) LD (c) empty (d) neither LI nor LD

(6) The vectors $(1+\alpha, 1-\alpha)$ and $(1-\alpha, 1+\alpha)$ of V_2 are LD only if $\alpha =$ ....

(a) 1 (b) 2 (c) -1 (d) 0

(7) $T : V_3 \to V_1$ defined by $T(x_1, x_2, x_3) =$ .... is not linear map .

(a) $x_1 + x_2 + x_3^2$ (b) $x_1 + x_3$ (c) $x_1 + x_2 + x_3$ (d) $x_1 - x_2$

(8) $\{x^2 - 1$, $x + 1$, ...$\}$ is basis for P_2 .

(a) $1 - x^2$ (b) $x^2 + x$ (c) $x - 1$ (d) $x^2 - x - 2$

(9) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B_1 = B_2 = \{e_1, e_2, e_3\}$,then the linear map T such that $A = (T : B_1, B_2)$ is given by T(x , y , z) = ...

(a) (x , 0 , 0) (b) (0 , y , 0) (c) (0 , 0 , z) (d) (x , y , z)

(10) If $T : P_3 \to P_2$ is linear map , then the matrix $(T : B_1, B_2)$ is of order ....

(a) 3×2 (b) 2×3 (c) 4×3 (d) 3×4

Que.2 Write TRUE or FALSE. [8]

(1) $\{1\}$ is a subspace of vector space V .

(2) In any vector space V , $\alpha\, \bar{0} = \bar{0}$

(3) Every superset of LD set is LD .

(4) $\{\sin^2 x$, $\cos 2x$, $1\}$ is LI set .

(P.T.O.)

(5) dim $P_3 = 4$

(6) Dimension of $\mathbb{C}$ over $\mathbb{R}$ is 1

(7) If $T : U \to V$ is linear map then $T(\alpha u_1 + \beta u_2) = \alpha T(u_1) + \beta T(u_2)$ for all scalar α, β , for all $u_1, u_2 \in U$.

(8) If $T : V_1 \to V_3$ defined by $T(x) = (x, 2x, 3x)$ then T is not linear .

Que.3 Answer the following (Any Ten) [20]

(1) Let v_1, v_2 be elements of a vector space V then prove that $[v_1, v_2] = [v_1 - v_2, v_1 + v_2]$.

(2) Is the set $\{(x_1, x_2, x_3) \in V_3 / \ x_3 = \sqrt{3}x_2\}$ subspaces of V_3 ? Verify it .

(3) Let S be a nonempty subset of a vector space V then prove that [S] is the smallest subspace of V containing S.

(4) Is the set $\{\ln x \ , \ \ln x^2 \ , \ \ln x^3\}$ LD ? Verify it .

(5) Determine a value of k that makes the vectors $\{(1, 2, k), (0, 1, k-1), (3, 4, 3)\}$ LD .

(6) If u , v , w are LI vectors in a vector space V then prove that $u+v, u-v, u-2v+w$ are also LI .

(7) Prove that $T : V_2 \to V_2$ defined by $T(x \ , \ y) = (x\cos\theta - y\sin\theta \ , \ x\sin\theta + y\cos\theta)$ linear .

(8) Is $T : P \to P$ defined by $T(p)(x) = 2 + 3x + 7x^2p(x)$ linear ? Verify it .

(9) Show that the set $\{(1, 1, 1), (1, -1, 1), (0, 1, 1)\}$ is a basis of V_3 .

(10) Let U and V be a vector space and $T : U \to V$ be any map such that $T(\alpha u_1 + u_2) = \alpha T(u_1) + T(u_2)$, $\forall$ scalar α , $\forall \ u_1, u_2 \in U$ then prove that T is linear .

(11) Determine a linear map $T : V_2 \to V_4$ such that $T(1, 1) = (0, 1, 0, 0)$, $T(1, -1) = (1, 0, 0, 0)$

(12) Let a linear map $T : V_3 \to V_2$ be defined by $T(x, y, z) = (x + y, y + z)$. Find $(T : B_1, B_2)$, where $B_1 = \{(1, 1, 1), (1, 0, 0), (1, 1, 0)\}$; $B_2 = \{e_1, e_2\}$

Que.4 Attempt the following (Any FOUR) [32]

(1) Let R^+ be the set of all positive real numbers .Define the operations as bellow : $u + v = uv$, $\forall \ u, v \ \in R^+$; $\alpha u = u^\alpha, \forall \ u \in R^+, \alpha \in \mathbb{R}$. Prove that R^+ is a real vector space .

(2) Let $S = \{x^3, x^2 + 2x, x^2 + 2, 1 - x\}$. Is $2x^3 + 3x^2 + 3x + 7 \in [S]$? Verify it .

(3) In a vector space V , suppose $S = \{v_1, v_2, ..., v_n\}$ is an ordered set of vectors with $v_1 \neq 0$ then prove that the set S is LD iff one of the vectors $v_1, v_2, ..., v_n$, say v_k, belongs to the span of $v_1, v_2, ..., v_{k-1}$.

(4) Which of the following sets are LD ? Verify it .

(i) $\{ \ x^2 - 4 \ , \ x + 2 \ , \ x - 2 \ , \ \frac{x^2}{3} \ \}$ in P_3 (ii) $\{ \ (1, 3, 2) \ , \ (1, -7, -8) \ , \ (2, 1, -1) \ \}$ in V_3

(5) If U and W are subspaces of a finite dimensional vector space V then prove that $dim(U + W) = dim(U) + dim(W) - dim(U \cap W)$.

(6) Determine whether the subset $S = \{ \ (1, 1, 1) \ , \ (1, 2, 3) \ , \ (-1, 0, 1) \ \}$ forms a basis for vector space V_3 ? If not , Determine the dimension of subspace [S] of V_3 .

(7) Let a linear map $T : P_2 \to P_3$ be defined by $T(P)(x) = xP(x)$. Find $(T : B_1, B_2)$, where $B_1 = \{1, 1 + x, 1 - x + x^2\}$; $B_2 = \{1 \ , \ 1 + x \ , \ x^2 \ , \ 2x - x^3\}$.

(8) Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{bmatrix}$. Determine a linear map $T : V_2 \to V_3$ such that $A = (T : B_1, B_2)$, where $B_1 = \{(1, 2), (-2, 1)\}$; $B_2 = \{(1, -1, -1), (1, 2, 3), (-1, 0, 2)\}$.

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