

SEAT No. _____



[8/A-7]

SARDAR PATEL UNIVERSITY (B.Sc. Sem.3 Examination)
MATHEMATICS - US03EMTH05 - Calculus and Algebra-1
1st October 2022, Saturday

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Time: 12:30 TO 02:30 p.m.

Maximum Marks: 70

Note: Figures to the right indicates the full marks.

Q.1 Answer the following by selecting the correct choice from the given options.

[10]

- $\lim_{x \rightarrow 0} (\cot x)^{\sin 2x}$ is of the form _____
(a) 0^0 (b) 0^∞ (c) ∞^0 (d) ∞^∞
- $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1}$ is of the form _____
(a) $\frac{\infty}{\infty}$ (b) $\frac{0}{0}$ (c) 1^∞ (d) ∞^0
- $u = \frac{x^2 y^2}{x+y}$ is a homogeneous function of degree _____
(a) 1 (b) 2 (c) 3 (d) 4
- If $u = x^2 + 2xy + y^2$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$ _____
(a) $2u$ (b) nu (c) zu (d) 0
- A matrix of order (3×3) has _____ elements
(a) 3 (b) 4 (c) 6 (d) 9
- A column matrix is of order _____
(a) $m \times n$ (b) $1 \times n$ (c) $m \times 1$ (d) 3×1
- In a Identity matrix, all the non-diagonal elements of the matrix are _____.
(a) 1 (b) \emptyset (c) i (d) 0
- Minor of a matrix is the _____ of a sub matrix.
(a) determinant (b) inverse (c) transpose (d) square
- Minor of a matrix is _____
(a) matrix (b) square matrix (c) sub-matrix (d) real number
- For matrix multiplication $A(BC) =$ _____
(a) $A(CB)$ (b) $(BA)C$ (c) $(AB)C$ (d) $B(AC)$

Q.2 Answer the given stamen is TRUE or FALSE

[08]

- $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\log(\cos x)}{\log(\cos 3x)}$ is an indeterminate form of $\frac{\infty}{\infty}$
- $\infty + \infty$ is an indeterminate form
- If $f(x, y) = x^2 y^2 + xy^3$ then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 4f(x, y)$
- If $z = x^3 - xy^4$ then $\frac{\partial^3 z}{\partial x^3} = 6$
- Null Matrix is always a square matrix.
- In equal matrices all the elements of a matrix are equal
- Determinant of a matrix is denoted by A'
- Number of elements in a matrix is greater than or equal to the number of elements in sub-matrix of that matrix.

Q-3 Answer ANY TEN of the following.

[20]

1. Evaluate $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$
2. Evaluate $\lim_{x \rightarrow 0} \frac{\log \sin x}{\cot x}$
3. Evaluate $\lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{(x - 2)}$
4. If $u = x^3 - 3xy^2$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
5. Check whether $z = ax^2 + 2hxy + by^2$ is homogeneous or not. If yes, find its degree.
6. If $f(x, y) = x \tan y + y \tan x$ then find f_{yx}
7. If A is Hermitian then prove that iA is skew-Hermitian
8. If A and B are symmetric matrices then prove that $A + B$ is symmetric
9. If $A = \begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$ then find AB
10. If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ then prove that $AA^T = I$
11. If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then prove that $A^2 - 4A + 5I = 0$
12. Find determinant of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Q-4 Answer ANY FOUR of the following.

[32]

1. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x}\right)^{\frac{5}{3x^2}}$
2. Evaluate $\lim_{x \rightarrow 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$
3. If $H = f(y - z, z - x, x - y)$ then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$
4. Verify Euler's theorem for $z = x^7 \log\left(\frac{y}{x}\right)$
5. If A and B are symmetric matrices of the same order then prove that AB is symmetric iff A and B are commutative.
6. Verify $(AB)^T = B^T A^T$ for $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix}$
7. State and prove Reversal law for the transpose of a product of two matrices.
8. Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

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