

SEAT No. _____



No. of Printed Pages: 3

[6/A-10]

SARDAR PATEL UNIVERSITY
B.Sc. Semester-III Examination
Friday, 30th September, 2022

Time:- 12:30 P.M. to 02:30 P.M.

Paper Code:- US03CSTA02

M.Marks:-70

Subject:-Elements of Probability Theory

Note:- Simple/Scientific calculators are allowed. Statistical Table is allowed.

Q.1. Multiple Choice Questions: -

[10]

- 1 For the events A and B if $A \subset B$ then the relation between $P(A)$ and $P(B)$ is _____.
(a) $P(A) \leq P(B)$ (b) $P(A) \geq P(B)$ (c) $P(A) \neq P(B)$ (d) none
- 2 Three coins are tossed, the number of sample points in a sample space is _____.
(a) 16 (b) 8 (c) 4 (d) none
- 3 The interval (Q_1, Q_3) includes _____ % of data.
(a) 25 (b) 50 (c) 75 (d) none
- 4 If $f(x) = \frac{x}{15}, x = 1, 2, 3, 4, 5$.
 $= 0$, otherwise. then $P(X > 3) =$ _____.
(a) $\frac{2}{15}$ (b) $\frac{3}{15}$ (c) $\frac{9}{15}$ (d) none
- 5 Let a random variable X denote the waiting time of a passenger at a railway station then X is _____ type of random variable.
(a) continuous (b) Discrete (c) complex (d) none.
- 6 If $M_X(t)$ is the m.g.f. of a random variable X and $Y = aX + b$ then $M_Y(t) =$ _____.
(a) $e^{bt} M_X(at)$ (b) $e^{at} M_X(bt)$ (c) $M_X(at+b)$ (d) none
- 7 If $P(t)$ is the p.g.f. of a discrete random variable X then $P(X = x) =$ _____.
(a) $\frac{dP(t)}{dt} |_{t=1}$ (b) $\frac{dP(t)}{dt} |_{t=0}$ (c) $\frac{dP(t)}{dt} |_{t=-1}$ (d) none
- 8 If X and Y are two independent random variables with $E(X) = 9$ and $E(X.Y) = 18$ then $E(Y) =$ _____.
(a) 6 (b) 4 (c) 2 (d) none
- 9 If two random variables X and Y are independent then $E(X+Y) =$ _____.
(a) $E(X) + E(Y)$ (b) $E(X).E(Y)$ (c) $E(X)/E(Y)$ (d) none
- 10 If $f(x,y) = 4xy, 0 < x < 1; 0 < y < 1$,
 $= 0$, elsewhere
is the joint p.d.f. of X and Y then $P(X < 0.50, Y < 0.50) =$ _____.
(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) none

[08]

Q.2. Fill in the blanks: -

1 The sum of all probabilities is equal to _____.

2 The value of $k =$ _____ for

x	1	2	3	4	5	6
f(x)	k	4k	10k	16k	25k	26k

3 If X and Y are two independent variables then $V(X-Y) =$ _____.

4 $P(X=x_i, Y=y_j) = P(X=x_i) * P(Y=y_j)$ for every i and j, then X and Y are _____.

State whether the statement is True or False.

5 The probability of throwing coin is $1/8$.

- 6 The probability generating function is given by $P_X(t) = \sum_x e^{xt} p(x)$.
 7 The m.g.f. of sum of two random variables is the product of their mgf's.
 8 For bi variate prob. distribution of X and Y $\sum_x \sum_y f(x,y) = 1$.

[20]

Q.3. Short Questions: - (Attempt any Ten)

- 1 Find the prob. of leap year, so at random leap year will contain 53 Sundays?
 2 A letter is chosen at random from the word 'TEACHER'. Find the prob. that letter is (i) a vowel (ii) a consonant.
 3 For any two events A and B, prove that If $B \subset A$, then $P(A \cap \bar{B}) = P(A) - P(B)$.
 4 The table below shows the prob. of X

x	1	2	3	4	5
p(x)	2k	3k	4k	5k	6k

- (i) For what value of k is the prob. is p.m.f or probability function.
 (ii) Find $P(3 \leq X \leq 5)$
 5 Consider the experiment of tossing of three fair coins. Let variable X denote the number heads. Find the probability mass function and cumulative distribution function.
 6 If $F(x) = 0, x < 1$
 $= \frac{x-1}{12}, 1 \leq x < 13$
 $= 1, x \geq 13$, is the c.d.f. of a random variable X then find the pdf of X and Q_1 , Median and P_{75} .
 7 Let X be a r.v. with the following prob. distribution:

x	-3	6	9
p(x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

- Find $E(x)$ and $E(x^2)$, Evaluate $E(2X+1)^2$.
 8 If $f(x) = \frac{x}{21}, x = 1, 2, 3, \dots, 6$.
 $= 0$, otherwise, is the pmf of a discrete r.v. X then find $E(X), V(X), E(2x+3)$.
 9 If X and Y are two r.v.'s then obtain $cov(ax, by)$.
 10 Is $f(x,y) = \frac{(2x+3y)}{120}, x=1, 2, 3; y=1, 2$
 $= 0$, otherwise, the joint p.m.f. of X and Y?
 11 If $f(x,y) = k(3x+2y), x=1, 2, 3, 4; y=1, 2, 3$
 $= 0$, otherwise is the joint p.m.f. of X and Y.
 Find (i) k (ii) the marginal distribution of X and Y.
 12 The joint p.d.f. of a two dimensional random variables (X,Y) is given by
 $f(x,y) = 2; 0 < x < 1, 0 < y < x$
 $= 0$, otherwise
 Find the marginal density function of X and Y.

[32]

Q.4. Long Questions: - (Attempt any four)

- 1 (i) For a post three persons A, B and C in the interview. The prob. of A be selected is twice that of B and that of B being selected is thrice that of C. What are the individual prob. of A, B and C being selected?

(ii) The prob. that a person will get an electric contract is $\frac{2}{5}$ and the prob. that he will not get plumbing contract is $\frac{4}{7}$. If the prob. of getting at least one contract is $\frac{2}{3}$. What is the prob. that he will get both contracts?

2 State and prove Bayes' Theorem.

There will be three candidates for the position of principal- Mr.X, Mr.Y and Mr.Z- whose chance of getting the appointment are in proportion 4:2:3 respectively. The probability that Mr. X if selected would introduce co-education in the college is 0.3. The probability of Mr. Y and Mr. Z doing the same are respectively 0.5 and 0.8.

(i) What is probability that there will be co-education in the college.

(ii) If there is co-education in the college, what is the probability that Mr. Z is the principal?

3 If $f(x) = kx, 0 < x < 2$

$$= k(5-x), 2 < x < 5$$

$= 0$, otherwise is the pdf of x then find

(i) k (ii) $P(1 < x \leq 2)$ (iii) $P(1 \leq x \leq 3)$ (iv) $P(x \geq 4)$. (v) cdf of $f(x)$

(vi) $P(3 < x < 4)$

4 A fair coin is tossed three times

(a) List the eight equally likely events

(b) If X represents the number of heads shown, determine $P(X=2)$.

(c) Find the prob. distribution of X .

(d) $P(X \leq 2)$

(e) Find $P(X \leq 2)$ and $P(X \leq 1/X \leq 2)$.

5 If $f(x) = 5e^{-5x}, 0 < x < \infty$

$= 0$, otherwise is the pdf of a rv X then find mgf, cgf and β_1, β_2

6 Two numbers are selected at random, without replacement from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the mean, variance, μ'_3 and μ'_4, β_1 and β_2 .

7 If X and Y are two random variables having joint density function

$$f(x,y) = \frac{1}{8} (6-x-y); 0 \leq x < 2, 2 \leq y < 4$$

$=$ otherwise

Find (i) $P(X < 1 \cap Y < 3)$ (ii) $P(X+Y < 3)$ (iii) $P(X < 1/Y < 3)$

8 State the condition for independent for two variables X and Y .

$$f(x,y) = \frac{xy}{36}, x = 1,2,3; y=1,2,3$$

$= 0$, otherwise is the joint pmf of X and Y .

(i) Show that X and Y are independent. (ii) $P(X < 2, Y < 3)$ (iii)

$P(X < 3, Y < 2)$ (iv) $P(X < 2/Y < 2)$.

(v) Marginal p.d.f. of X and Y variables.
