[19] SEAT No.________ No. of Printed Pages: 02

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B.Sc. Sem:III

Subject : Mathematics US03CMTH01 [Advanced Calculus] Max.Marks: 70

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Q.1 Choose the correct option for each of the following. [10]

(1) $\int_0^1 \int_0^2 dxdy =$ ......

(a) 1 (b) 0 (c) 3 (d) 2

(2) If $x = r\cos\theta, y = r\sin\theta$ then Jacobian J=

(a) r (b) 1 (c) r^2 (d) None of these

(3) In usual notations, I_0 = ...

(a) $I_X + I_Y$ (b) $I_X - I_Y$ (c) $I_X I_Y$ (d) None of these

(4) In double integral ,Area of region R is given by A = ...

(a) $\iint_R dx\, dy$ (b) $\iint_C dx\, dy$ (c) $\iint_R f(x,y)dx\, dy$ (d) None of these

(5) $\int_C [fdx + gdy + hdz]$ is independent of path iff fdx+gdy+hdz is ......

(a) 0 (b) not exact (c) 1 (d) exact

(6) If f = xy^2, g = $-x^2y$ then $\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}$ = ...

(a) $4xy$ (b) $-4xy$ (c) $2xy$ (d) $-2xy$

(7) Moment of inertia of surface S about x-axis is denoted by ......

(a) I_x (b) I_y (c) Iy (d) Ix

(8) Parametric form of $x^2 + y^2 = z^2$ is $\bar{r}$ = ...

(a) ucosv$\bar{i}$ +usinv$\bar{j}$ + u$\bar{k}$ (b) ucosu$\bar{i}$ +vsinv$\bar{j}$ + u$\bar{k}$

(c) ucosv$\bar{i}$ +usinu$\bar{j}$ + v$\bar{k}$ (d) cosv$\bar{i}$ +vsinv$\bar{j}$ + u$\bar{k}$

(9) If f is harmonic function, then ....

(a) $\nabla^2 f = 0$ (b) $\bar{\nabla}^2 f = 0$ (c) $\nabla f = 0$ (d) $\bar{\nabla} f = 0$

(10) The unit normal vector to the surface f(x,y,z) = 0 denoted by ....

(a) n (b) N (c) $\bar{n}$ (d) $\bar{N}$

Q.2 Do as directed **[08]**

(1) True or False : $\frac{ds}{dt} = \left|\frac{d\bar{r}}{dt}\right|$.

(2) For x + y =u ,x-y =v then Jacobian J =.......

(3) True or False: The differential form $xdx + ydy + zdz$ is exact.

(4) True or False: The area of a region $r = a(1 + \cos\theta)$ is $\frac{\pi a^2}{2}$.

(5) The parametric form of a plane $y = x$ is ....

P.T.O.

(6) If $W = 2x^2 + y^2$ then $\nabla^2 W =$ ....

(7) $\int_0^1 \int_0^1 \int_0^1 x\, dxdydz$ = ...

(8) True or False : If $\bar{n} = \bar{k}$ then $dA = dxdz$

Q.3 Attempt any TEN **[20]**

(1) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^1 x^2y^2 dxdy$.

(2) Evaluate the line integral $\int_C (3x^2 + 3y^2)\, ds$, where

C: over the path y = x from (0,0) to (1,1) (Counterclockwise direction).

(3) Define: Line integral.

(4) Find area of plane region in polar form.

(5) Change the order of integration in $\int_0^c \int_0^y f(x,y)dxdy$.

(6) When a line integral is said to be independent of path.

(7) Write down the parametric form of a surface.

(8) Obtain first fundamental form of a surface in polar form.

(9) By using divergence theorem , evaluate $\iint_S e^y dzdx$,

where S: $0 \leq x \leq 3, 0 \leq y \leq 2, 0 \leq z \leq 1$.

(10) If f is harmonic then porve that $\iint_S \frac{\partial f}{\partial n} dA = 0$.

(11) State 2^{nd} form of Green's theorem.

(12) Prove first form of Green's theorem

Q.4 Attempt any Four. **[32]**

(1) Evaluate the integral $\int_C (y^2dx - x^2dy)$, where

C: along circle $x^2 + y^2 = 1$ from (0,1) to (1,0) (counter clockwise).

(2) Evaluate $\iint_R (x^2 + y^2)\, dxdy$, where R is the parallelogram with vertices :

(0,0),(1,1),(2,0),(1,-1),& $x + y = u, x - y = v$.

(3) State and Prove Green's theorem for plane.

(4) Change the order of integration in $\int_0^{a\cos\alpha} \int_{x\tan\alpha}^{\sqrt{a^2-x^2}} f(x,y)dxdy$.

(5) State and Prove divergence theorem of Gauss.

(6) Find area of the surface $z = x^2 + y^2$,where $0 \leq z \leq b$.

(7) State and Prove Stoke's theorem.

(8) By using triple integral, find the total mass of a mass distribution of density $\sigma = xy$ in a region R : The tetrahedral with vertices (0,0,0), (1,0,0), (0,1,0) and (0,0,1).

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